ALEKS MATH FOR BEGINNERS

The Ultimate Step by Step Guide to Preparing for the ALEKS Math Test

By

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Thank you for choosing Effortless Math for your ALEKS Math test preparation and congratulations on making the decision to take the ALEKS test! It’s a remarkable move you are taking, one that shouldn’t be diminished in any capacity. That’s why you need to use every tool possible to ensure you succeed on the test with the highest possible score, and this extensive study guide is one such tool.

If math has never been a strong subject for you, don’t worry! This book will help you prepare for (and even ACE) the ALEKS Math assessment. As test day draws nearer, effective preparation becomes increasingly more important. Thankfully, you have this comprehensive study guide to help you get ready for the test. With this guide, you can feel confident that you will be more than ready for the ALEKS Math test when the time comes.

First and foremost, it is important to note that this book is a study guide and not a textbook. It is best read from cover to cover. Every lesson of this "self-guided math book" was carefully developed to ensure that you are making the most effective use of your time while preparing for the test. This up-to-date guide reflects the 2021 test guidelines and will put you on the right track to hone your math skills, overcome exam anxiety, and boost your confidence, so that you can have your best to succeed on the ALEKS Math test.
This study guide will:

☑ Explain the format of the ALEKS Math test.

☑ Describe specific test-taking strategies that you can use on the test.

☑ Provide ALEKS Math test-taking tips.

☑ Review all ALEKS Math concepts and topics you will be tested on.

☑ Help you identify the areas in which you need to concentrate your study time.

☑ Offer exercises that help you develop the basic math skills you will learn in each section.

☑ Give 2 realistic and full-length practice tests (featuring new question types) with detailed answers to help you measure your exam readiness and build confidence.

This resource contains everything you will ever need to succeed on the ALEKS Math test. You’ll get in-depth instructions on every math topic as well as tips and techniques on how to answer each question type. You’ll also get plenty of practice questions to boost your test-taking confidence.

In addition, in the following pages you’ll find:

➢ **How to Use This Book Effectively** – This section provides you with step-by-step instructions on how to get the most out of this comprehensive study guide.

➢ **How to study for the ALEKS Math Test** – A six-step study program has been developed to help you make the best use of this book and prepare for your ALEKS Math test. Here you’ll find tips and strategies to guide your study program and help you understand ALEKS Math and how to ace the test.

➢ **ALEKS Math Review** – Learn everything you need to know about the ALEKS Math test.

➢ **ALEKS Math Test-Taking Strategies** – Learn how to effectively put these recommended test-taking techniques into use for improving your ALEKS Math score.

➢ **Test Day Tips** – Review these tips to make sure you will do your best when the big day comes.

### Effortless Math’s ALEKS Online Center

Effortless Math Online ALEKS Center offers a complete study program, including the following:

- ✔ Step-by-step instructions on how to prepare for the ALEKS Math test
- ✔ Numerous ALEKS Math worksheets to help you measure your math skills
- ✔ Complete list of ALEKS Math formulas
- ✔ Video lessons for all ALEKS Math topics
- ✔ Full-length ALEKS Math practice tests
- ✔ And much more...

No Registration Required.

Visit [EffortlessMath.com/ALEKS](https://www.EffortlessMath.com/ALEKS) to find your online ALEKS Math resources.
Look no further when you need a study guide to improve your math skills to succeed on the math portion of the ALEKS test. Each chapter of this comprehensive guide to the ALEKS Math will provide you with the knowledge, tools, and understanding needed for every topic covered on the test.

It’s imperative that you understand each topic before moving onto another one, as that’s the way to guarantee your success. Each chapter provides you with examples and a step-by-step guide of every concept to better understand the content that will be on the test. To get the best possible results from this book:

➢ **Begin studying long before your test date.** This provides you ample time to learn the different math concepts. The earlier you begin studying for the test, the sharper your skills will be. Do not procrastinate! Provide yourself with plenty of time to learn the concepts and feel comfortable that you understand them when your test date arrives.

➢ **Practice consistently.** Study ALEKS Math concepts at least 20 to 30 minutes a day. Remember, slow and steady wins the race, which can be applied to preparing for the ALEKS Math test. Instead of cramming to tackle everything at once, be patient and learn the math topics in short bursts.

➢ **Whenever you get a math problem wrong, mark it off, and review it later** to make sure you understand the concept.

➢ **Start each session by looking over the previous related material.**

➢ **Once you’ve reviewed the book’s lessons, take a practice test** at the back of the book to gauge your level of readiness. Then, review your results. Read detailed answers and solutions for each question you missed.

➢ **Take another practice test** to get an idea of how ready you are to take the actual exam. Taking the practice tests will give you the confidence you need on test day. Simulate the ALEKS testing environment by sitting in a quiet room free from distraction. Make sure to clock yourself with a timer.
How to Study for the ALEKS Math Test

Studying for the ALEKS Math test can be a really daunting and boring task. What’s the best way to go about it? Is there a certain study method that works better than others? Well, studying for the ALEKS Math can be done effectively. The following six-step program has been designed to make preparing for the ALEKS Math test more efficient and less overwhelming.

**Step 1**: Create a study plan
**Step 2**: Choose your study resources
**Step 3**: Review, Learn, Practice
**Step 4**: Learn and practice test-taking strategies
**Step 5**: Learn the ALEKS Test format and take practice tests
**Step 6**: Analyze your performance

**STEP 1: Create a Study Plan**

It’s always easier to get things done when you have a plan. Creating a study plan for the ALEKS Math test can help you to stay on track with your studies. It’s important to sit down and prepare a study plan with what works with your life, work, and any other obligations you may have. Devote enough time each day to studying. It’s also a great idea to break down each section of the exam into blocks and study one concept at a time.

It’s important to understand that there is no “right” way to create a study plan. Your study plan will be personalized based on your specific needs and learning style.

Follow these guidelines to create an effective study plan for your ALEKS Math test:

- **Analyze your learning style and study habits** – Everyone has a different learning style. It is essential to embrace your individuality and the unique way you learn. Think about what works and what doesn’t work for you. Do you prefer ALEKS Math prep books or a combination of textbooks and video lessons? Does it work better for you if you study every night for thirty minutes or is it more effective to study in the morning before going to work?
★ **Evaluate your schedule** – Review your current schedule and find out how much time you can consistently devote to ALEKS Math study.

★ **Develop a schedule** – Now it’s time to add your study schedule to your calendar like any other obligation. Schedule time for study, practice, and review. Plan out which topic you will study on which day to ensure that you’re devoting enough time to each concept. Develop a study plan that is mindful, realistic, and flexible.

★ **Stick to your schedule** – A study plan is only effective when it is followed consistently. You should try to develop a study plan that you can follow for the length of your study program.

★ **Evaluate your study plan and adjust as needed** – Sometimes you need to adjust your plan when you have new commitments. Check in with yourself regularly to make sure that you’re not falling behind in your study plan. Remember, the most important thing is sticking to your plan. Your study plan is all about helping you be more productive. If you find that your study plan is not as effective as you want, don't get discouraged. It's okay to make changes as you figure out what works best for you.

**STEP 2: Choose Your Study Resources**

There are numerous textbooks and online resources available for the ALEKS Math test, and it may not be clear where to begin. Don’t worry! This study guide provides everything you need to fully prepare for your ALEKS Math test. In addition to the book content, you can also use Effortless Math's online resources. (video lessons, worksheets, formulas, etc.)

Simply visit [EffortlessMath.com/ALEKS](http://EffortlessMath.com/ALEKS) to find your online ALEKS Math resources.
**STEP 3: Review, Learn, Practice**

This ALEKS Math study guide breaks down each subject into specific skills or content areas. For instance, the percent concept is separated into different topics—percent calculation, percent increase and decrease, percent problems, etc. Use this book to help you go over all key math concepts and topics on the ALEKS Math test.

As you read each chapter, take notes or highlight the concepts you would like to go over again in the future. If you're unfamiliar with a topic or something is difficult for you, do additional research on it. For each math topic, plenty of instructions, step-by-step guides, and examples are provided to ensure you get a good grasp of the material. You can also find video lessons on the Effortless Math website for each ALEKS Math concept.

Quickly review the topics you do understand to get a brush-up of the material. Be sure to do the practice questions provided at the end of every chapter to measure your understanding of the concepts.

**STEP 4: Learn and Practice Test-taking Strategies**

In the following sections, you will find important test-taking strategies and tips that can help you earn extra points. You'll learn how to think strategically and when to guess if you don’t know the answer to a question. Using ALEKS Math test-taking strategies and tips can help you raise your score and do well on the test. Apply test taking strategies on the practice tests to help you boost your confidence.
**STEP 5: Learn the ALEKS Test Format and Take Practice Tests**

The *ALEKS Test Review* section provides information about the structure of the ALEKS test. Read this section to learn more about the ALEKS test structure, different test sections, the number of questions in each section, and the section time limits. When you have a prior understanding of the test format and different types of ALEKS Math questions, you'll feel more confident when you take the actual exam.

Once you have read through the instructions and lessons and feel like you are ready to go – take advantage of both of the full-length ALEKS Math practice tests available in this study guide. Use the practice tests to sharpen your skills and build confidence.

The ALEKS Math practice tests offered at the end of the book are formatted similarly to the actual ALEKS Math test. When you take each practice test, try to simulate actual testing conditions. To take the practice tests, sit in a quiet space, time yourself, and work through as many of the questions as time allows. The practice tests are followed by detailed answer explanations to help you find your weak areas, learn from your mistakes, and raise your ALEKS Math score.

**STEP 6: Analyze Your Performance**

After taking the practice tests, look over the answer keys and explanations to learn which questions you answered correctly and which you did not. Never be discouraged if you make a few mistakes. See them as a learning opportunity. This will highlight your strengths and weaknesses.

You can use the results to determine if you need additional practice or if you are ready to take the actual ALEKS Math test.
Looking for more?


Or scan this QR code.

No Registration Required.
ALEKS Test Review

ALEKS (Assessment and Learning in Knowledge Spaces) is an artificial intelligence-based assessment tool to measure the strengths and weaknesses of a student’s mathematical knowledge. ALEKS is available for a variety of subjects and courses in K-12, Higher Education, and Continuing Education. The findings of ALEKS’s assessment test help to find an appropriate level for course placement. The ALEKS math placement assessment ensures students’ readiness for particular math courses at colleges.

ALEKS does not use multiple-choice questions like most other standardized tests. Instead, it utilizes adaptable and easy-to-use method that mimic paper and pencil techniques. When taking the ALEKS test, a brief tutorial helps you learn how to use ALEKS answer input tools. You then begin the ALEKS Assessment. In about 30 to 45 minutes, the test measures your current content knowledge by asking 20 to 30 questions. ALEKS is a Computer Adaptive (CA) assessment. It means that each question will be chosen on the basis of answers to all the previous questions. Therefore, each set of assessment questions is unique. The ALEKS Math assessment does not allow you to use a personal calculator. But for some questions ALEKS onscreen calculator button is active and the test taker can use it.

Key Features of the ALEKS Mathematics Assessment

Some key features of the ALEKS Math assessment are:

❖ Mathematics questions on ALEKS are adaptive to identify the student’s knowledge from a comprehensive standard curriculum, ranging from basic arithmetic up to precalculus, including trigonometry but not calculus.

❖ Unlike other standardized tests, the ALEKS assessment does not provide a "grade" or "raw score." Instead, ALEKS identifies which concepts the student has mastered and what topics the student needs to learn.

❖ ALEKS does not use multiple-choice questions. Instead, students need to produce authentic mathematical input.

❖ There is no time limit for taking the ALEKS Math assessment. But it usually takes 30 to 45 minutes to complete the assessment.
The ALEKS Math score is between 1 and 100 and is interpreted as a percentage correct. A higher ALEKS score indicates that the test-taker has mastered more math concepts. ALEKS Math assessment tool evaluates mastery of a comprehensive set of mathematics skills ranging from basic arithmetic up to precalculus, including trigonometry but not calculus. It will place students in classes up to Calculus.
ALEKS Math Test-Taking Strategies

Here are some test-taking strategies that you can use to maximize your performance and results on the ALEKS Math test.

**#1: Use This Approach to Answer Every ALEKS Math Question**

- Review the question to identify keywords and important information.
- Translate the keywords into math operations so you can solve the problem.
- Review the answer choices. What are the differences between answer choices?
- Draw or label a diagram if needed.
- Try to find patterns.
- Find the right method to answer the question. Use straightforward math, plug in numbers, or test the answer choices (backsolving).
- Double-check your work.

**#2: Answer Every ALEKS Math Question**

Don’t leave any fields empty! ALEKS is a Computer Adaptive (CA) assessment. Therefore, you cannot leave a question unanswered and you cannot go back to previous questions.

Even if you’re unable to work out a problem, strive to answer it. Take a guess if you have to. You will not lose points by getting an answer wrong, though you may gain a point by getting it correct!
#3: BALLPARK

A ballpark answer is a rough approximation. When we become overwhelmed by calculations and figures, we end up making silly mistakes. A decimal that is moved by one unit can change an answer from right to wrong, regardless of the number of steps that you went through to get it. That’s where ballparking can play a big part.

If you think you know what the correct answer may be (even if it’s just a ballpark answer), you’ll usually have the ability to estimate the range of possible answers and avoid simple mistakes.

#4: PLUGGING IN NUMBERS

“Plugging in numbers” is a strategy that can be applied to a wide range of different math problems on the ALEKS Math test. This approach is typically used to simplify a challenging question so that it is more understandable. By using the strategy carefully, you can find the answer without too much trouble.

The concept is fairly straightforward—replace unknown variables in a problem with certain values. When selecting a number, consider the following:

- Choose a number that’s basic (just not too basic). Generally, you should avoid choosing 1 (or even 0). A decent choice is 2.
- Try not to choose a number that is displayed in the problem.
- Make sure you keep your numbers different if you need to choose at least two of them.
- If your question contains fractions, then a potential right answer may involve either an LCD (least common denominator) or an LCD multiple.
- 100 is the number you should choose when you are dealing with problems involving percentages.
ALEKS Mathematics – Test Day Tips

After practicing and reviewing all the math concepts you’ve been taught, and taking some ALEKS mathematics practice tests, you’ll be prepared for test day. Consider the following tips to be extra-ready come test time.

Before Your Test

What to do the night before:

- **Relax!** One day before your test, study lightly or skip studying altogether. You shouldn’t attempt to learn something new, either. There are plenty of reasons why studying the evening before a big test can work against you. Put it this way—a marathoner wouldn’t go out for a sprint before the day of a big race. Mental marathoners—such as yourself—should not study for any more than one hour 24 hours before a ALEKS test. That’s because your brain requires some rest to be at its best. The night before your exam, spend some time with family or friends, or read a book.

- **Avoid bright screens** - You’ll have to get some good shuteye the night before your test. Bright screens (such as the ones coming from your laptop, TV, or mobile device) should be avoided altogether. Staring at such a screen will keep your brain up, making it hard to drift asleep at a reasonable hour.

- **Make sure your dinner is healthy** - The meal that you have for dinner should be nutritious. Be sure to drink plenty of water as well. Load up on your complex carbohydrates, much like a marathon runner would do. Pasta, rice, and potatoes are ideal options here, as are vegetables and protein sources.

- **Get your bag ready for test day** - Prefer to take ALEKS in the Testing Office? The night prior to your test, pack your bag with your stationery, admissions pass, ID, and any other gear that you need. Keep the bag right by your front door. If you prefer to take the test at home, find a quite place without any distractions.

- **Make plans to reach the testing site** - If you are taking the test at the testing office, ensure that you understand precisely how you will arrive at the site of the test. If parking is something you’ll have to find first, plan for it. If you’re dependent on public transit, then review the schedule. You should also make sure that the train/bus/subway/streetcar you use will be running. Find out about road closures as well. If a parent or friend is accompanying you, ensure that they understand what steps they have to take as well.
The Day of the Test

- **Get up reasonably early, but not too early.**

- **Have breakfast** - Breakfast improves your concentration, memory, and mood. As such, make sure the breakfast that you eat in the morning is healthy. The last thing you want to be is distracted by a grumbling tummy. If it’s not your own stomach making those noises, another test taker close to you might be instead. Prevent discomfort or embarrassment by consuming a healthy breakfast. Bring a snack with you if you think you’ll need it.

- **Follow your daily routine** - Do you watch TV in the morning while getting ready for the day? Don’t break your usual habits on the day of the test. Likewise, if coffee isn’t something you drink in the morning, then don’t take it up the habit hours before your test. Routine consistency lets you concentrate on the main objective—doing the best you can on your test.

- **Wear layers** - Dress yourself up in comfortable layers if you are taking the test at the testing site. You should be ready for any kind of internal temperature. If it gets too warm during the test, take a layer off.

- **Make your voice heard** - If something is off, speak to a proctor. If medical attention is needed or if you’ll require anything, consult the proctor prior to the start of the test. Any doubts you have should be clarified. You should be entering the test site with a state of mind that is completely clear.

- **Have faith in yourself** - When you feel confident, you will be able to perform at your best. When you are waiting for the test to begin, envision yourself receiving an outstanding result. Try to see yourself as someone who knows all the answers, no matter what the questions are. A lot of athletes tend to use this technique—particularly before a big competition. Your expectations will be reflected by your performance.
**During your test**

- **Be calm and breathe deeply** - You need to relax before the test, and some deep breathing will go a long way to help you do that. Be confident and calm. You got this. Everybody feels a little stressed out just before an evaluation of any kind is set to begin. Learn some effective breathing exercises. Spend a minute meditating before the test starts. Filter out any negative thoughts you have. Exhibit confidence when having such thoughts.

- **Concentrate on the test** - Refrain from comparing yourself to anyone else. You shouldn't be distracted by the people near you or random noise. Concentrate exclusively on the test. If you find yourself irritated by surrounding noises, earplugs can be used to block sounds off close to you. Don’t forget—the test is going to last an hour or more. Some of that time will be dedicated to brief sections. Concentrate on the specific section you are working on during a particular moment. Do not let your mind wander off to upcoming or previous questions.

- **Try to answer each question individually** - Focus only on the question you are working on. Use one of the test-taking strategies to solve the problem. If you aren’t able to come up with an answer, don’t get frustrated. Simply guess, then move onto the next question.

- **Don’t forget to breathe!** Whenever you notice your mind wandering, your stress levels boosting, or frustration brewing, take a thirty-second break. Shut your eyes, drop your pencil, breathe deeply, and let your shoulders relax. You will end up being more productive when you allow yourself to relax for a moment.

**After your test**

- **Take it easy** - You will need to set some time aside to relax and decompress once the test has concluded. There is no need to stress yourself out about what you could’ve said, or what you may have done wrong. At this point, there’s nothing you can do about it. Your energy and time would be better spent on something that will bring you happiness for the remainder of your day.
Redoing the test - Did you succeed on the test? Congratulations! Your hard work paid off! Succeeding on this test means that you are now ready to take college level courses.

If you didn't receive the result you expected, though, don't worry! The test can be retaken. In such cases, you will need to follow the retake policy. You also need to re-register to take the exam again.
# Contents

Chapter: **Fractions and Mixed Numbers**

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Simplifying Fractions</td>
<td>2</td>
</tr>
<tr>
<td>Adding and Subtracting Fractions</td>
<td>3</td>
</tr>
<tr>
<td>Multiplying and Dividing Fractions</td>
<td>4</td>
</tr>
<tr>
<td>Adding Mixed Numbers</td>
<td>5</td>
</tr>
<tr>
<td>Subtract Mixed Numbers</td>
<td>6</td>
</tr>
<tr>
<td>Multiplying Mixed Numbers</td>
<td>7</td>
</tr>
<tr>
<td>Dividing Mixed Numbers</td>
<td>8</td>
</tr>
<tr>
<td>Chapter 1: Practices</td>
<td>9</td>
</tr>
<tr>
<td>Chapter 1: Answers</td>
<td>12</td>
</tr>
</tbody>
</table>

Chapter: **Decimals**

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Comparing Decimals</td>
<td>14</td>
</tr>
<tr>
<td>Rounding Decimals</td>
<td>15</td>
</tr>
<tr>
<td>Adding and Subtracting Decimals</td>
<td>16</td>
</tr>
<tr>
<td>Multiplying and Dividing Decimals</td>
<td>17</td>
</tr>
<tr>
<td>Chapter 2: Practices</td>
<td>18</td>
</tr>
<tr>
<td>Chapter 2: Answers</td>
<td>20</td>
</tr>
</tbody>
</table>

Chapter: **Integers and Order of Operations**

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Adding and Subtracting Integers</td>
<td>22</td>
</tr>
<tr>
<td>Multiplying and Dividing Integers</td>
<td>23</td>
</tr>
<tr>
<td>Order of Operations</td>
<td>24</td>
</tr>
<tr>
<td>Integers and Absolute Value</td>
<td>25</td>
</tr>
<tr>
<td>Chapter 3: Practices</td>
<td>26</td>
</tr>
<tr>
<td>Chapter 3: Answers</td>
<td>28</td>
</tr>
</tbody>
</table>

Chapter: **Ratios and Proportions**

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Simplifying Ratios</td>
<td>30</td>
</tr>
<tr>
<td>Proportional Ratios</td>
<td>31</td>
</tr>
<tr>
<td>Similarity and Ratios</td>
<td>32</td>
</tr>
<tr>
<td>Chapter 4: Practices</td>
<td>33</td>
</tr>
<tr>
<td>Chapter 4: Answers</td>
<td>36</td>
</tr>
</tbody>
</table>

Chapter: **Percentage**

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Percent Problems</td>
<td>38</td>
</tr>
<tr>
<td>Percent of Increase and Decrease</td>
<td>39</td>
</tr>
<tr>
<td>Discount, Tax and Tip</td>
<td>40</td>
</tr>
<tr>
<td>Simple Interest</td>
<td>41</td>
</tr>
<tr>
<td>Chapter 5: Practices</td>
<td>42</td>
</tr>
<tr>
<td>Chapter 5: Answers</td>
<td>44</td>
</tr>
</tbody>
</table>
## Contents

### Chapter: **Exponents and Variables** 45

- Multiplication Property of Exponents ........................................ 46
- Division Property of Exponents .............................................. 47
- Powers of Products and Quotients ............................................ 48
- Zero and Negative Exponents .................................................. 49
- Negative Exponents and Negative Bases .................................... 50
- Scientific Notation ............................................................... 51
- Radicals ................................................................................. 52
- Chapter 6: Practices .............................................................. 53
- Chapter 6: Answers ............................................................... 56

### Chapter: **Expressions and Variables** 57

- Simplifying Variable Expressions .............................................. 58
- Simplifying Polynomial Expressions ......................................... 59
- The Distributive Property ....................................................... 60
- Evaluating One Variable ....................................................... 61
- Evaluating Two Variables ...................................................... 62
- Chapter 7: Practices .............................................................. 63
- Chapter 7: Answers ............................................................... 66

### Chapter: **Equations and Inequalities** 67

- One–Step Equations ............................................................... 68
- Multi–Step Equations ............................................................ 69
- System of Equations ............................................................. 70
- Graphing Single–Variable Inequalities ...................................... 71
- One–Step Inequalities ............................................................ 72
- Multi–Step Inequalities .......................................................... 73
- Chapter 8: Practices .............................................................. 74
- Chapter 8: Answers ............................................................... 76

### Chapter: **Lines and Slope** 77

- Finding Slope ......................................................................... 78
- Graphing Lines Using Slope–Intercept Form ................................ 79
- Writing Linear Equations ....................................................... 80
- Finding Midpoint ................................................................. 81
- Finding Distance of Two Points ............................................. 82
- Graphing Linear Inequalities .................................................. 83
- Chapter 9: Practices .............................................................. 84
- Chapter 9: Answers ............................................................... 86
## Contents

### Chapter: **Polynomials**

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Simplifying Polynomials</td>
<td>88</td>
</tr>
<tr>
<td>Adding and Subtracting Polynomials</td>
<td>89</td>
</tr>
<tr>
<td>Multiplying Monomials</td>
<td>90</td>
</tr>
<tr>
<td>Multiplying and Dividing Monomials</td>
<td>91</td>
</tr>
<tr>
<td>Multiplying a Polynomial and a Monomial</td>
<td>92</td>
</tr>
<tr>
<td>Multiplying Binomials</td>
<td>93</td>
</tr>
<tr>
<td>Factoring Trinomials</td>
<td>94</td>
</tr>
<tr>
<td>Chapter 10: Practices</td>
<td>95</td>
</tr>
<tr>
<td>Chapter 10: Answers</td>
<td>98</td>
</tr>
</tbody>
</table>

### Chapter: **Geometry and Solid Figures**

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>The Pythagorean Theorem</td>
<td>100</td>
</tr>
<tr>
<td>Complementary and Supplementary angles</td>
<td>101</td>
</tr>
<tr>
<td>Parallel lines and Transversals</td>
<td>102</td>
</tr>
<tr>
<td>Triangles</td>
<td>103</td>
</tr>
<tr>
<td>Special Right Triangles</td>
<td>104</td>
</tr>
<tr>
<td>Polygons</td>
<td>105</td>
</tr>
<tr>
<td>Circles</td>
<td>106</td>
</tr>
<tr>
<td>Trapezoids</td>
<td>107</td>
</tr>
<tr>
<td>Cubes</td>
<td>108</td>
</tr>
<tr>
<td>Rectangular Prisms</td>
<td>109</td>
</tr>
<tr>
<td>Cylinder</td>
<td>110</td>
</tr>
<tr>
<td>Chapter 11: Practices</td>
<td>111</td>
</tr>
<tr>
<td>Chapter 11: Answers</td>
<td>114</td>
</tr>
</tbody>
</table>

### Chapter: **Statistics**

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean, Median, Mode, and Range of the Given Data</td>
<td>116</td>
</tr>
<tr>
<td>Pie Graph</td>
<td>117</td>
</tr>
<tr>
<td>Probability Problems</td>
<td>118</td>
</tr>
<tr>
<td>Permutations and Combinations</td>
<td>119</td>
</tr>
<tr>
<td>Chapter 12: Practices</td>
<td>120</td>
</tr>
<tr>
<td>Chapter 12: Answers</td>
<td>122</td>
</tr>
</tbody>
</table>

### Chapter: **Functions Operations**

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Function Notation and Evaluation</td>
<td>124</td>
</tr>
<tr>
<td>Adding and Subtracting Functions</td>
<td>125</td>
</tr>
<tr>
<td>Multiplying and Dividing Functions</td>
<td>126</td>
</tr>
<tr>
<td>Composition of Functions</td>
<td>127</td>
</tr>
<tr>
<td>Function Inverses</td>
<td>128</td>
</tr>
<tr>
<td>Chapter 13: Practices</td>
<td>129</td>
</tr>
<tr>
<td>Chapter 13: Answers</td>
<td>132</td>
</tr>
</tbody>
</table>
Chapter: Quadratic 133
14
Solving a Quadratic Equation ............................................. 134
Graphing Quadratic Functions ........................................... 135
Solving Quadratic Inequalities .......................................... 136
Graphing Quadratic Inequalities ........................................ 137
Chapter 14: Practices ....................................................... 138
Chapter 14: Answers ....................................................... 140

Chapter: Complex Numbers 143
15
Adding and Subtracting Complex Numbers ......................... 144
Multiplying and Dividing Complex Numbers ....................... 145
Rationalizing Imaginary Denominators ............................... 146
Chapter 15: Practices ....................................................... 147
Answers – Chapter 15 ....................................................... 148

Chapter: Radicals 149
16
Simplifying Radical Expressions ........................................ 150
Adding and Subtracting Radical Expressions ....................... 151
Multiplying Radical Expressions ....................................... 152
Rationalizing Radical Expressions .................................... 153
Radical Equations ........................................................... 154
Domain and Range of Radical Functions ............................ 155
Chapter 16: Practices ....................................................... 156
Answers – Chapter 16 ....................................................... 158

Chapter: Logarithms 159
17
Evaluating Logarithms ...................................................... 160
Expanding and Condensing Logarithms ............................... 161
Natural Logarithms .......................................................... 161
Solving Logarithmic Equations ......................................... 163
Chapter 17: Practices ....................................................... 163
Answers – Chapter 17 ....................................................... 166

Chapter: Circles 167
18
Circumference and Area of Circles .................................... 168
Arc Length and Sector Area ............................................... 169
Equation of a Circle ........................................................ 170
Finding the Center and the Radius of Circles ....................... 171
Chapter 18: Practices ....................................................... 172
Answers – Chapter 18 ....................................................... 174
Math topics that you’ll learn in this chapter:

- Simplifying Fractions
- Adding and Subtracting Fractions
- Multiplying and Dividing Fractions
- Adding Mixed Numbers
- Subtracting Mixed Numbers
- Multiplying Mixed Numbers
- Dividing Mixed Numbers
Simplifying Fractions

- A fraction contains two numbers separated by a bar between them. The bottom number, called the denominator, is the total number of equally divided portions in one whole. The top number, called the numerator, is how many portions you have. And the bar represents the operation of division.

- Simplifying a fraction means reducing it to the lowest terms. To simplify a fraction, evenly divide both the top and bottom of the fraction by $2, 3, 5, 7, \ldots$ etc.

- Continue until you can't go any further.

Examples:

Example 1. Simplify $\frac{18}{30}$

Solution: To simplify $\frac{18}{30}$, find a number that both 18 and 30 are divisible by. Both are divisible by 6. Then: $\frac{18}{30} = \frac{18\div6}{30\div6} = \frac{3}{5}$

Example 2. Simplify $\frac{48}{80}$

Solution: To simplify $\frac{48}{80}$, find a number that both 48 and 80 are divisible by. Both are divisible by 8 and 16. Then: $\frac{48}{80} = \frac{48\div8}{80\div8} = \frac{6}{10}$, 6 and 10 are divisible by 2, then: $\frac{6}{10} = \frac{3}{5}$ or $\frac{48}{80} = \frac{48\div16}{80\div16} = \frac{3}{5}$

Example 3. Simplify $\frac{40}{120}$

Solution: To simplify $\frac{40}{120}$, find a number that both 40 and 120 are divisible by. Both are divisible by 40, then: $\frac{40}{120} = \frac{40\div40}{120\div40} = \frac{1}{3}$
Adding and Subtracting Fractions

- For “like” fractions (fractions with the same denominator), add or subtract the numerators (top numbers) and write the answer over the common denominator (bottom numbers).

- Adding and Subtracting fractions with the same denominator:

\[
\frac{a}{b} + \frac{c}{b} = \frac{a+c}{b} \quad \frac{a}{b} - \frac{c}{b} = \frac{a-c}{b}
\]

- Find equivalent fractions with the same denominator before you can add or subtract fractions with different denominators.

- Adding and Subtracting fractions with different denominators:

\[
\frac{a}{b} + \frac{c}{d} = \frac{ad+bc}{bd} \quad \frac{a}{b} - \frac{c}{d} = \frac{ad-cb}{bd}
\]

Examples:

Example 1. Find the sum \(\frac{2}{3} + \frac{1}{2} = \) \(\frac{7}{6}\)

Solution: These two fractions are “unlike” fractions. (they have different denominators). Use this formula:

\[
\frac{a}{b} + \frac{c}{d} = \frac{ad+bc}{bd}
\]

Then:

\[
\frac{2}{3} + \frac{1}{2} = \frac{(2)(2)+(3)(1)}{3 \times 2} = \frac{4+3}{6} = \frac{7}{6}
\]

Example 2. Find the difference \(\frac{3}{5} - \frac{2}{7} = \)

Solution: For “unlike” fractions, find equivalent fractions with the same denominator before you can add or subtract fractions with different denominators. Use this formula:

\[
\frac{a}{b} - \frac{c}{d} = \frac{ad-cb}{bd}
\]

Then:

\[
\frac{3}{5} - \frac{2}{7} = \frac{(3)(7)-(2)(5)}{5 \times 7} = \frac{21-10}{35} = \frac{11}{35}
\]
Multiplying and Dividing Fractions

- Multiplying fractions: multiply the top numbers and multiply the bottom numbers. Simplify if necessary. \( \frac{a}{b} \times \frac{c}{d} = \frac{a \times c}{b \times d} \)

- Dividing fractions: Keep, Change, Flip

- Keep the first fraction, change the division sign to multiplication, and flip the numerator and denominator of the second fraction. Then, solve!

\[
\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c} = \frac{a \times d}{b \times c}
\]

Examples:

Example 1. Multiply. \( \frac{2}{3} \times \frac{3}{5} = \)

**Solution:** Multiply the top numbers and multiply the bottom numbers.

\[
\frac{2}{3} \times \frac{3}{5} = \frac{2 \times 3}{3 \times 5} = \frac{6}{15}
\]

Example 2. Solve. \( \frac{3}{4} \div \frac{2}{5} = \)

**Solution:** Keep the first fraction, change the division sign to multiplication, and flip the numerator and denominator of the second fraction.

Then: \( \frac{3}{4} \div \frac{2}{5} = \frac{3}{4} \times \frac{5}{2} = \frac{3 \times 5}{4 \times 2} = \frac{15}{8} \)

Example 3. Calculate. \( \frac{4}{5} \times \frac{3}{4} = \)

**Solution:** Multiply the top numbers and multiply the bottom numbers.

\[
\frac{4}{5} \times \frac{3}{4} = \frac{4 \times 3}{5 \times 4} = \frac{12}{20}, \text{ simplify: } \frac{12}{20} = \frac{12+4}{20+4} = \frac{3}{5}
\]

Example 4. Solve. \( \frac{5}{6} \div \frac{3}{7} = \)

**Solution:** Keep the first fraction, change the division sign to multiplication, and flip the numerator and denominator of the second fraction.

Then: \( \frac{5}{6} \div \frac{3}{7} = \frac{5 \times 7}{6 \times 3} = \frac{35}{18} \)
Chapter 1: Fractions and Mixed Numbers

Adding Mixed Numbers

Use the following steps for adding mixed numbers:

- Add whole numbers of the mixed numbers.
- Add the fractions of the mixed numbers.
- Find the Least Common Denominator (LCD) if necessary.
- Add whole numbers and fractions.
- Write your answer in lowest terms.

Examples:

Example 1. Add mixed numbers. \(2\frac{1}{2} + 1\frac{2}{3} =\)

Solution: Let’s rewriting our equation with parts separated, \(2\frac{1}{2} + 1\frac{2}{3} = 2 + \frac{1}{2} + 1 + \frac{2}{3}\). Now, add whole number parts: \(2 + 1 = 3\)

Add the fraction parts \(\frac{1}{2} + \frac{2}{3}\). Rewrite to solve with the equivalent fractions. \(\frac{1}{2} + \frac{2}{3} = \frac{3}{6} + \frac{4}{6} = \frac{7}{6}\). The answer is an improper fraction (numerator is bigger than denominator). Convert the improper fraction into a mixed number: \(\frac{7}{6} = 1\frac{1}{6}\). Now, combine the whole and fraction parts: \(3 + 1\frac{1}{6} = 4\frac{1}{6}\)

Example 2. Find the sum. \(1\frac{3}{4} + 2\frac{1}{2} =\)

Solution: Rewriting our equation with parts separated, \(1 + \frac{3}{4} + 2 + \frac{1}{2}\). Add the whole number parts:

\(1 + 2 = 3\). Add the fraction parts: \(\frac{3}{4} + \frac{1}{2} = \frac{3}{4} + \frac{2}{4} = \frac{5}{4}\)

Convert the improper fraction into a mixed number: \(\frac{5}{4} = 1\frac{1}{4}\).

Now, combine the whole and fraction parts: \(3 + 1\frac{1}{4} = 4\frac{1}{4}\)
Subtracting Mixed Numbers

Use these steps for subtracting mixed numbers.

- Convert mixed numbers into improper fractions. \( \frac{a}{b} + \frac{c}{d} = \frac{ad + bc}{bd} \)
- Find equivalent fractions with the same denominator for unlike fractions. (fractions with different denominators)
- Subtract the second fraction from the first one. \( \frac{a}{b} - \frac{c}{d} = \frac{ad - bc}{bd} \)
- Write your answer in lowest terms.
- If the answer is an improper fraction, convert it into a mixed number.

Examples:

Example 1. Subtract. \( 2\frac{1}{3} - 1\frac{1}{2} = \)

Solution: Convert mixed numbers into fractions: \( 2\frac{1}{3} = \frac{2 \times 3 + 1}{3} = \frac{7}{3} \) and \( 1\frac{1}{2} = \frac{1 \times 2 + 1}{2} = \frac{3}{2} \)

These two fractions are “unlike” fractions. (they have different denominators). Find equivalent fractions with the same denominator. Use this formula: \( \frac{a}{b} - \frac{c}{d} = \frac{ad - bc}{bd} \)

\[
\frac{7}{3} - \frac{3}{2} = \frac{(7)(2) - (3)(3)}{3 \times 2} = \frac{14 - 9}{6} = \frac{5}{6}
\]

Example 2. Subtract. \( 3\frac{4}{7} - 2\frac{3}{4} = \)

Solution: Convert mixed numbers into fractions: \( 3\frac{4}{7} = \frac{3 \times 7 + 4}{7} = \frac{25}{7} \) and \( 2\frac{3}{4} = \frac{2 \times 4 + 3}{4} = \frac{11}{4} \)

Then: \( 3\frac{4}{7} - 2\frac{3}{4} = \frac{25}{7} - \frac{11}{4} = \frac{(25)(4) - (11)(7)}{7 \times 4} = \frac{23}{28} \)
Chapter 1: Fractions and Mixed Numbers

Multiplying Mixed Numbers

Use the following steps for multiplying mixed numbers:

- Convert the mixed numbers into fractions. $a \frac{c}{b} = a + \frac{c}{b} = \frac{ab + c}{b}$

- Multiply fractions. $\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}$

- Write your answer in lowest terms.

- If the answer is an improper fraction (numerator is bigger than denominator), convert it into a mixed number.

Examples:

Example 1. Multiply. $4 \frac{1}{2} \times 2 \frac{2}{5} =$

Solution: Convert mixed numbers into fractions, $4 \frac{1}{2} = \frac{4 \times 2 + 1}{2} = \frac{9}{2}$ and 
$2 \frac{2}{5} = \frac{2 \times 5 + 2}{5} = \frac{12}{5}$

Apply the fractions rule for multiplication, $\frac{9}{2} \times \frac{12}{5} = \frac{9 \times 12}{2 \times 5} = \frac{108}{10}$

The answer is an improper fraction. Convert it into a mixed number. $\frac{108}{10} = 10 \frac{4}{5}$

Example 2. Multiply. $3 \frac{2}{3} \times 2 \frac{5}{6} =$

Solution: Converting mixed numbers into fractions, $3 \frac{2}{3} = \frac{3 \times 3 + 2}{3} = \frac{11}{3}$ and $2 \frac{5}{6} = \frac{2 \times 6 + 5}{6} = \frac{17}{6}$

Apply the fractions rule for multiplication, $\frac{11}{3} \times \frac{17}{6} = \frac{11 \times 17}{3 \times 6} = \frac{187}{18} = 10 \frac{7}{18}$

Example 3. Multiply mixed numbers. $5 \frac{1}{4} \times 3 \frac{3}{8} =$

Solution: Converting mixed numbers to fractions, $5 \frac{1}{4} = \frac{21}{4}$ and $3 \frac{3}{8} = \frac{27}{8}$. Multiply two fractions:

$\frac{21}{4} \times \frac{27}{8} = \frac{21 \times 27}{4 \times 8} = \frac{567}{32} = 17 \frac{23}{32}$
Dividing Mixed Numbers

Use the following steps for dividing mixed numbers:

- Convert the mixed numbers into fractions. \( \frac{a}{b} \times \frac{c}{d} = \frac{a c}{b d} \)
- Divide fractions: Keep, Change, Flip: Keep the first fraction, change the division sign to multiplication, and flip the numerator and denominator of the second fraction. Then, solve \( \frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c} = \frac{a d}{b c} \)
- Write your answer in lowest terms.
- If the answer is an improper fraction (numerator is bigger than denominator), convert it into a mixed number.

Examples:

Example 1. Solve. \( 2 \frac{1}{3} \div 1 \frac{1}{2} \)

Solution: Convert mixed numbers into fractions: \( 2 \frac{1}{3} = \frac{2 \times 3 + 1}{3} = \frac{7}{3} \) and \( 1 \frac{1}{2} = \frac{1 \times 2 + 1}{2} = \frac{3}{2} \)

Keep, Change, Flip: \( \frac{7}{3} \div \frac{3}{2} = \frac{7}{3} \times \frac{2}{3} = \frac{7 	imes 2}{3 \times 3} = \frac{14}{9} \). The answer is an improper fraction.

Convert it into a mixed number: \( \frac{14}{9} = 1 \frac{5}{9} \)

Example 2. Solve. \( 3 \frac{3}{4} \div 2 \frac{2}{5} \)

Solution: Convert mixed numbers to fractions, then solve:
\[
3 \frac{3}{4} \div 2 \frac{2}{5} = \frac{15}{4} \div \frac{12}{5} = \frac{15}{4} \times \frac{5}{12} = \frac{75}{48} = 1 \frac{9}{16}
\]

Example 3. Solve. \( 2 \frac{4}{5} \div 1 \frac{2}{3} \)

Solution: Converting mixed numbers to fractions: \( 2 \frac{4}{5} = \frac{14}{5} \) and \( 1 \frac{2}{3} = \frac{5}{3} \)

Keep, Change, Flip: \( \frac{14}{5} \div \frac{5}{3} = \frac{14}{5} \times \frac{3}{5} = \frac{14 \times 3}{5 \times 5} = \frac{42}{25} = 1 \frac{17}{25} \)
Chapter 1: Practices

❖ Simplify each fraction.

1) \( \frac{2}{8} = \)

5) \( \frac{25}{45} = \)

2) \( \frac{5}{15} = \)

6) \( \frac{42}{54} = \)

3) \( \frac{10}{90} = \)

7) \( \frac{48}{60} = \)

4) \( \frac{12}{16} = \)

8) \( \frac{52}{169} = \)

❖ Find the sum or difference.

9) \( \frac{3}{10} + \frac{2}{10} = \)

13) \( \frac{7}{54} - \frac{1}{9} = \)

10) \( \frac{4}{9} - \frac{1}{9} = \)

14) \( \frac{4}{5} - \frac{1}{6} = \)

11) \( \frac{2}{3} + \frac{6}{15} = \)

15) \( \frac{6}{7} - \frac{3}{8} = \)

12) \( \frac{17}{24} - \frac{5}{8} = \)

16) \( \frac{2}{13} + \frac{1}{4} = \)

❖ Find the products or quotients.

17) \( \frac{2}{9} \div \frac{4}{3} = \)

19) \( \frac{9}{25} \times \frac{5}{27} = \)

18) \( \frac{14}{5} \div \frac{28}{35} = \)

20) \( \frac{65}{72} \times \frac{12}{15} = \)

❖ Find the sum.

21) \( 2\frac{1}{5} + 1\frac{2}{5} = \)

24) \( 2\frac{2}{7} + 4\frac{1}{21} = \)

22) \( 5\frac{1}{9} + 2\frac{7}{9} = \)

25) \( 5\frac{3}{5} + 1\frac{4}{9} = \)

23) \( 2\frac{3}{4} + 1\frac{1}{8} = \)

26) \( 3\frac{3}{11} + 4\frac{6}{7} = \)

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Find the difference.

27) \( \frac{5}{3} - \frac{2}{3} = \)
28) \( \frac{4}{7} - \frac{1}{3} = \)
29) \( \frac{3}{10} - \frac{2}{9} = \)
30) \( \frac{1}{2} - \frac{3}{1} = \)
31) \( \frac{3}{4} - \frac{1}{28} = \)
32) \( \frac{2}{7} - \frac{3}{6} = \)
33) \( \frac{3}{10} - \frac{3}{4} = \)
34) \( \frac{9}{20} - \frac{1}{3} = \)

Find the products.

35) \( \frac{1}{2} \times \frac{3}{7} = \)
36) \( \frac{4}{5} \times \frac{1}{3} = \)
37) \( \frac{4}{2} \times \frac{5}{6} = \)
38) \( \frac{7}{2} \times \frac{1}{5} = \)
39) \( \frac{1}{2} \times \frac{3}{2} = \)
40) \( \frac{1}{5} \times \frac{4}{5} = \)
41) \( \frac{1}{2} \times \frac{4}{3} = \)
42) \( \frac{9}{10} \times \frac{4}{2} = \)

Solve.

43) \( \frac{1}{3} \div \frac{2}{3} = \)
44) \( \frac{1}{4} \div \frac{1}{2} = \)
45) \( \frac{5}{1} \div \frac{1}{2} = \)
46) \( \frac{3}{7} \div \frac{1}{8} = \)
47) \( \frac{1}{5} \div \frac{2}{3} = \)
48) \( \frac{2}{3} \div \frac{1}{3} = \)
49) \( \frac{4}{1} \div \frac{2}{3} = \)
50) \( \frac{2}{11} \div \frac{1}{8} = \)
Chapter 1: Answers

1) \( \frac{1}{4} \)  
2) \( \frac{1}{3} \)  
3) \( \frac{1}{9} \)  
4) \( \frac{3}{4} \)  
5) \( \frac{5}{9} \)  
6) \( \frac{7}{9} \)  
7) \( \frac{4}{5} \)  
8) \( \frac{4}{13} \)  
9) \( \frac{1}{2} \)  
10) \( \frac{1}{3} \)  
11) \( \frac{16}{15} = 1 \frac{1}{15} \)  
12) \( \frac{1}{12} \)  
13) \( \frac{1}{54} \)  
14) \( \frac{19}{30} \)  
15) \( \frac{27}{56} \)  
16) \( \frac{21}{52} \)  
17) \( \frac{1}{6} \)  
18) \( \frac{7}{2} = 3 \frac{1}{2} \)  
19) \( \frac{1}{15} \)  
20) \( \frac{13}{18} \)  
21) \( 3 \frac{3}{5} \)  
22) \( 7 \frac{8}{9} \)  
23) \( 3 \frac{7}{8} \)  
24) \( 6 \frac{1}{3} \)  
25) \( 7 \frac{2}{45} \)  
26) \( 8 \frac{10}{77} \)  
27) \( 2 \frac{2}{3} \)  
28) \( 3 \frac{2}{5} \)  
29) \( 1 \frac{1}{9} \)  
30) \( 3 \frac{1}{6} \)  
31) \( 2 \frac{5}{7} \)  
32) \( 1 \frac{5}{42} \)  
33) \( 1 \frac{11}{20} \)  
34) \( 4 \frac{7}{60} \)  
35) \( 3 \frac{9}{14} \)  
36) \( 2 \frac{4}{5} \)  
37) \( 8 \frac{1}{4} \)  
38) \( 4 \frac{4}{35} \)  
39) \( 12 \frac{1}{10} \)  
40) \( 12 \)  
41) \( 14 \frac{2}{5} \)  
42) \( 22 \frac{1}{20} \)  
43) \( 4 \frac{5}{5} \)  
44) \( 1 \frac{1}{2} \)  
45) \( 1 \frac{11}{21} \)  
46) \( 2 \frac{58}{63} \)  
47) \( 1 \frac{23}{40} \)  
48) \( 1 \frac{7}{33} \)  
49) \( 1 \frac{11}{16} \)  
50) \( 1 \frac{5}{99} \)  

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Math topics that you’ll learn in this chapter:

- Comparing Decimals
- Rounding Decimals
- Adding and Subtracting Decimals
- Multiplying and Dividing Decimals
Comparing Decimals

- A decimal is a fraction written in a special form. For example, instead of writing $\frac{1}{2}$, you can write 0.5

- A Decimal Number contains a Decimal Point. It separates the whole number part from the fractional part of a decimal number.

- Let's review decimal place values: Example: 53.9861
  5: tens  3: ones  9: tenths
  8: hundredths  6: thousandths  1: ten thousandths

- To compare decimals, compare each digit of two decimals in the same place value. Start from left. Compare hundreds, tens, ones, tenth, hundredth, etc.

- To compare numbers, use these symbols:
  Equal to $=$,  Less than $<$,  Greater than $>$
  Greater than or equal $\geq$,  Less than or equal $\leq$

Examples:

Example 1. Compare 0.03 and 0.30.

Solution: 0.30 is greater than 0.03, because the tenth place of 0.30 is 3, but the tenth place of 0.03 is zero. Then: $0.03 < 0.30$

Example 2. Compare 0.0217 and 0.217.

Solution: 0.217 is greater than 0.0217, because the tenth place of 0.217 is 2, but the tenth place of 0.0217 is zero. Then: $0.0217 < 0.217$
Rounding Decimals

- We can round decimals to a certain accuracy or number of decimal places. This is used to make calculations easier to do and results easier to understand when exact values are not too important.

- First, you'll need to remember your place values: For example: 12.4869
  1: tens
  2: ones
  4: tenths
  8: hundredths
  6: thousandths
  9: tens thousandths

- To round a decimal, first find the place value you'll round to.

- Find the digit to the right of the place value you're rounding to. If it is 5 or bigger, add 1 to the place value you're rounding to and remove all digits on its right side. If the digit to the right of the place value is less than 5, keep the place value and remove all digits on the right.

Examples:

Example 1. Round 4.3679 to the thousandth place value.

Solution: First, look at the next place value to the right, (tens thousandths). It’s 9 and it is greater than 5. Thus add 1 to the digit in the thousandth place. The thousandth place is 7. \( \rightarrow 7 + 1 = 8 \), then,
The answer is 4.368

Example 2. Round 1.5237 to the nearest hundredth.

Solution: First, look at the digit to the right of hundredth (thousandths place value). It’s 3 and it is less than 5, thus remove all the digits to the right of hundredth place. Then, the answer is 1.52
Adding and Subtracting Decimals

- Line up the decimal numbers.
- Add zeros to have the same number of digits for both numbers if necessary.
- Remember your place values: For example: 73.5196
  
  7: tens  
  3: ones  
  5: tenths  
  1: hundredths  
  9: thousandths  
  6: tens thousandths
- Add or subtract using column addition or subtraction.

Examples:

Example 1. Add 1.7 + 4.12

Solution: First, line up the numbers: $\frac{17}{1.70} + \frac{4.12}{1.70} \rightarrow$ Add a zero to have the same number of digits for both numbers. $\frac{1.70}{2} + \frac{4.12}{2} \rightarrow$ Start with the hundredths place: $0 + 2 = 2$, $\frac{1.70}{1.70} + \frac{4.12}{1.70} \rightarrow$ Continue with tenths place: $7 + 1 = 8$, $\frac{4.12}{.82} \rightarrow$ Add the ones place: $4 + 1 = 5$, $\frac{4.12}{5.82}$

Example 2. Find the difference 5.58 − 4.23

Solution: First, line up the numbers: $\frac{5.58}{5.58} \rightarrow$ Start with the hundredths place: $8 - 3 = 5$, $\frac{4.23}{5} \rightarrow$ Continue with tenths place. $5 - 2 = 3$, $\frac{4.23}{.35} \rightarrow$ Subtract the ones place. $5 - 4 = 1$, $\frac{4.23}{1.35}$
Multiplying and Dividing Decimals

For multiplying decimals:

- Ignore the decimal point and set up and multiply the numbers as you do with whole numbers.
- Count the total number of decimal places in both of the factors.
- Place the decimal point in the product.

For dividing decimals:

- If the divisor is not a whole number, move the decimal point to the right to make it a whole number. Do the same for the dividend.
- Divide similar to whole numbers.

Examples:

Example 1. Find the product. \(0.65 \times 0.24 =\)

**Solution:** Set up and multiply the numbers as you do with whole numbers. Line up the numbers: \(\frac{65}{24}\) → Start with the ones place then continue with other digits \(\frac{65 \times 24}{1,560}\). Count the total number of decimal places in both of the factors. There are four decimals digits. (two for each factor 0.65 and 0.24) Then: \(0.65 \times 0.24 = 0.1560\)

Example 2. Find the quotient. \(1.20 \div 0.4 =\)

**Solution:** The divisor is not a whole number. Multiply it by 10 to get 4: \(\rightarrow 0.4 \times 10 = 4\)
Do the same for the dividend to get 12. \(\rightarrow 1.20 \times 10 = 12\)
Now, divide \(12 \div 4 = 3\). The answer is 3.
Chapter 2: Practices

Compare. Use >, =, and <

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Round each decimal to the nearest whole number.

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Find the sum or difference.

31) 12.1 + 36.2 =  
32) 56.3 − 22.2 =  
33) 45.1 + 12.8 =  
34) 27.9 − 16.4 =  
35) 98.8 − 56.6 =  
36) 28.45 + 13.22 =  
37) 16.78 + 45.11 =  
38) 86.16 − 72.12 =  
39) 96.23 − 28.32 =  
40) 57.33 + 67.46 =  
41) 46.26 − 39.49 =  
42) 44.95 + 76.53 =  
43) 79.37 − 52.89 =  
44) 19.99 + 28.7 =  
45) 83.48 − 49.3 =  
46) 19.6 + 42.98 =  

Find the product or quotient.

47) 3.3 × 0.2 =  
48) 2.4 ÷ 0.3 =  
49) 8.1 × 1.4 =  
50) 4.8 ÷ 0.2 =  
51) 4.1 × 0.3 =  
52) 8.6 ÷ 0.2 =  
53) 9.9 × 0.8 =  
54) 1.84 ÷ 0.2 =  
55) 2.1 × 8.4 =  
56) 1.6 × 4.5 =  
57) 9.2 × 3.1 =  
58) 36.6 ÷ 1.6 =  
59) 1.91 × 5.2 =  
60) 3.65 × 1.4 =  
61) 24.82 ÷ 0.4 =  
62) 12.4 × 4.20 =
## Chapter 2: Answers

1) <   
2) >   
3) <   
4) <   
5) <   
6) >   
7) >   
8) <   
9) <   
10) <  
11) =  
12) =  
13) =  
14) <  
15) 6  
16) 6  
17) 12 
18) 9  
19) 8  
20) 22 
21) 7  
22) 16 
23) 13 
24) 17 
25) 68 
26) 43 
27) 56 
28) 14 
29) 79 
30) 98 
31) 48.3
32) 34.1
33) 57.9
34) 11.5
35) 42.2
36) 41.67
37) 61.89
38) 14.04
39) 67.91
40) 124.79
41) 6.77
42) 121.48
43) 26.48
44) 48.69
45) 34.18
46) 62.58
47) 0.66
48) 8
49) 11.34
50) 24
51) 1.23
52) 43
53) 7.92
54) 9.2
55) 17.64
56) 7.2
57) 28.52
58) 22.875
59) 9.932
60) 5.11
61) 62.05
62) 52.08

CHAPTER

3

Integers and Order of Operations

Math topics that you’ll learn in this chapter:

☑ Adding and Subtracting Integers
☑ Multiplying and Dividing Integers
☑ Order of Operations
☑ Integers and Absolute Value
Adding and Subtracting Integers

- Integers include zero, counting numbers, and the negative of the counting numbers. \{…, -3, -2, -1, 0, 1, 2, 3, …\}
- Add a positive integer by moving to the right on the number line. (you will get a bigger number)
- Add a negative integer by moving to the left on the number line. (you will get a smaller number)
- Subtract an integer by adding its opposite.

Examples:

Example 1. Solve. \((-2) - (-8) =\)

Solution: Keep the first number and convert the sign of the second number to its opposite. (change subtraction into addition. Then: \((-2) + 8 = 6\)

Example 2. Solve. \(4 + (5 - 10) =\)

Solution: First, subtract the numbers in brackets, \(5 - 10 = -5\).
Then: \(4 + (-5) = \rightarrow \text{change addition into subtraction: } 4 - 5 = -1\)

Example 3. Solve. \((9 - 14) + 15 =\)

Solution: First, subtract the numbers in brackets, \(9 - 14 = -5\)
Then: \(-5 + 15 = \rightarrow -5 + 15 = 10\)

Example 4. Solve. \(12 + (-3 - 10) =\)

Solution: First, subtract the numbers in brackets, \(-3 - 10 = -13\)
Then: \(12 + (-13) = \rightarrow \text{change addition into subtraction: } 12 - 13 = -1\)
Multiplying and Dividing Integers

Use the following rules for multiplying and dividing integers:

- (negative) × (negative) = positive
- (negative) ÷ (negative) = positive
- (negative) × (positive) = negative
- (negative) ÷ (positive) = negative
- (positive) × (positive) = positive
- (positive) ÷ (negative) = negative

Examples:

Example 1. Solve. \(3 \times (-4) =\)

Solution: Use this rule: (positive) × (negative) = negative.
Then: \((3) \times (-4) = -12\)

Example 2. Solve. \((-3) + (-24 ÷ 3) =\)

Solution: First, divide \(-24\) by 3; the numbers in brackets, use this rule:
(negative) ÷ (positive) = negative. Then: \(-24 ÷ 3 = -8\)
\((-3) + (-24 ÷ 3) = (-3) + (-8) = -3 - 8 = -11\)

Example 3. Solve. \((12 - 15) \times (-2) =\)

Solution: First, subtract the numbers in brackets,
\(12 - 15 = -3 \rightarrow (-3) \times (-2) =\)
Now use this rule: (negative) × (negative) = positive \(\rightarrow (-3) \times (-2) = 6\)

Example 4. Solve. \((12 - 8) ÷ (-4) =\)

Solution: First, subtract the numbers in brackets,
\(12 - 8 = 4 \rightarrow (4) ÷ (-4) =\)
Now use this rule: (positive) ÷ (negative) = negative \(\rightarrow (4) ÷ (-4) = -1\)
Order of Operations

- In Mathematics, “operations” are addition, subtraction, multiplication, division, exponentiation (written as \( b^n \)), and grouping.

- When there is more than one math operation in an expression, use PEMDAS: (to memorize this rule, remember the phrase “Please Excuse My Dear Aunt Sally”.)

  ❖ Parentheses
  ❖ Exponents
  ❖ Multiplication and Division (from left to right)
  ❖ Addition and Subtraction (from left to right)

Examples:

**Example 1.** Calculate. \((2 + 6) ÷ (2^2 ÷ 4) = \)

*Solution:* First, simplify inside parentheses:
\((8) ÷ (4 ÷ 4) = (8) ÷ (1), \) Then: \((8) ÷ (1) = 8\)

**Example 2.** Solve. \((6 \times 5) - (14 - 5) = \)

*Solution:* First, calculate within parentheses: \((6 \times 5) - (14 - 5) = (30) - (9), \) Then: \((30) - (9) = 21\)

**Example 3.** Calculate. \(-4[(3 \times 6) ÷ (3^2 \times 2)] = \)

*Solution:* First, calculate within parentheses:
\(-4[(18) ÷ (9 \times 2)] = -4[(18) ÷ (18)] = -4[1] \) multiply \(-4 \) and 1. Then: \(-4[1] = -4\)

**Example 4.** Solve. \((28 ÷ 7) + (-19 + 3) = \)

*Solution:* First, calculate within parentheses:
\((28 ÷ 7) + (-19 + 3) = (4) + (-16) \) Then: \((4) - (16) = -12\)
Integers and Absolute Value

- The absolute value of a number is its distance from zero, in either direction, on the number line. For example, the distance of 9 and \(-9\) from zero on number line is 9.

- The absolute value of an integer is the numerical value without its sign (negative or positive).

- The vertical bar is used for absolute value as in \(|x|\).

- The absolute value of a number is never negative; because it only shows "how far the number is from zero".

Examples:

Example 1. Calculate. \(|14 - 2| \times 5 =\)

Solution: First, solve \(|14 - 2|, \rightarrow |14 - 2| = |12|, the absolute value of 12 is 12, \(|12| = 12\) Then: \(12 \times 5 = 60\)

Example 2. Solve. \(\frac{|-24|}{4} \times |5 - 7| =\)

Solution: First, find \(|-24|, \rightarrow\) the absolute value of \(-24\) is 24, Then: \(|-24| = 24, \frac{24}{4} \times |5 - 7| =\)
Now, calculate \(|5 - 7|, \rightarrow |5 - 7| = |-2|, the absolute value of \(-2\) is 2. \(|-2| = 2\) then: \(\frac{24}{4} \times 2 = 6 \times 2 = 12\)

Example 3. Solve. \(|8 - 2| \times \frac{|-4 \times 7|}{2} =\)

Solution: First, calculate \(|8 - 2|, \rightarrow |8 - 2| = |6|, the absolute value of 6 is 6, \(|6| = 6\) Then: \(6 \times \frac{|-4 \times 7|}{2}\)
Now calculate \(|-4 \times 7|, \rightarrow |-4 \times 7| = |-28|, the absolute value of \(-28\) is 28, \(|-28| = 28\) Then: \(6 \times \frac{28}{2} = 6 \times 14 = 84\)
Chapter 3: Practices

Find each sum or difference.

1) \(-9 + 16 = \)
2) \(-18 - 6 = \)
3) \(-24 + 10 = \)
4) \(30 + (-5) = \)
5) \(15 + (-3) = \)
6) \((-13) + (-4) = \)
7) \(25 + (3 - 10) = \)
8) \(12 - (-6 + 9) = \)
9) \(5 - (-2 + 7) = \)
10) \((-11) + (-5 + 6) = \)
11) \((-3) + (9 - 16) = \)
12) \((-8) - (13 + 4) = \)
13) \((-7 + 9) - 39 = \)
14) \((-30 + 6) - 14 = \)
15) \((-5 + 9) + (-3 + 7) = \)
16) \((8 - 19) - (-4 + 12) = \)
17) \((-9 + 2) - (6 - 7) = \)
18) \((-12 - 5) - (-4 - 14) = \)

Solve.

19) \(3 \times (-6) = \)
20) \((-32) \div 4 = \)
21) \((-5) \times 4 = \)
22) \((25) \div (-5) = \)
23) \((-72) \div 8 = \)
24) \((-2) \times (-6) \times 5 = \)
25) \((-2) \times 3 \times (-7) = \)
26) \((-1) \times (-3) \times (-5) = \)
27) \((-2) \times (-3) \times (-6) = \)
28) \((-12 + 3) \times (-5) = \)
29) \((-3 + 4) \times (-11) = \)
30) \((-9) \times (6 - 5) = \)
31) \((-3 - 7) \times (-6) = \)
32) \((-7 + 3) \times (-9 + 6) = \)
33) \((-15) \div (-17 + 12) = \)
34) \((-3 - 2) \times (-9 + 7) = \)
35) \((-15 + 31) \div (-2) = \)
36) \((-64) \div (-16 + 8) = \)
Evaluate each expression.

37) \(3 + (2 \times 5) = \)

38) \((5 \times 4) - 7 = \)

39) \((-9 \times 2) + 6 = \)

40) \((7 \times 3) - (-5) = \)

41) \((-8) + (2 \times 7) = \)

42) \((9 - 6) + (3 \times 4) = \)

43) \((-19 + 5) + (6 \times 2) = \)

44) \((32 \div 4) + (1 - 13) = \)

45) \((-36 \div 6) - (12 + 3) = \)

46) \((-16 + 5) - (54 \div 9) = \)

47) \((-20 + 4) - (35 \div 5) = \)

48) \((42 \div 7) + (2 \times 3) = \)

49) \((28 \div 4) + (2 \times 6) = \)

50) \(2[(3 \times 3) - (4 \times 5)] = \)

51) \(3[(2 \times 8) + (4 \times 3)] = \)

52) \(2[(9 \times 3) - (6 \times 4)] = \)

53) \(4[(4 \times 8) \div (4 \times 4)] = \)

54) \(-5[(10 \times 8) \div (5 \times 8)] = \)

Find the answers.

55) \(|-5| + |7 - 10| = \)

56) \(|-4 + 6| + |-2| = \)

57) \(|-9| + |1 - 9| = \)

58) \(|-7| - |8 - 12| = \)

59) \(|9 - 11| + |8 - 15| = \)

60) \(|-7 + 10| - |-8 + 3| = \)

61) \(|-12 + 6| - |3 - 9| = \)

62) \(5 + |2 - 6| + |3 - 4| = \)

63) \(-4 + |2 - 6| + |1 - 9| = \)

64) \(|-6| \times |-7| + |2 - 8| = \)

65) \(|-12| \times |-3| + |4 - 28| = \)

66) \(|4 \times (-2)| \times |-9| = \)

67) \(|-3 \times 2| \times |-5| = \)

68) \(|3 - 12| - |-3 \times 7| = \)

69) \(|-9| + |-7 \times 5| = \)

70) \(|-11| + |-6 \times 4| = \)

71) \(|-4 \times 2 + 6| \times |-2 \times 8| = \)

72) \(|-1 \times 5 + 2| \times |-4| = \)
### Chapter 3: Answers

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CHAPTER 4
Ratios and Proportions

Math topics that you’ll learn in this chapter:

☑ Simplifying Ratios
☑ Proportional Ratios
☑ Similarity and Ratios
Simplifying Ratios

- Ratios are used to make comparisons between two numbers.
- Ratios can be written as a fraction, using the word "to", or with a colon. Example: \( \frac{3}{4} \) or "3 to 4" or 3:4
- You can calculate equivalent ratios by multiplying or dividing both sides of the ratio by the same number.

Examples:

Example 1. Simplify. 8:2 =

Solution: Both numbers 8 and 2 are divisible by 2 \( \Rightarrow 8 \div 2 = 4 \),
4 \( \div 2 = 2 \), Then: 8:2 = 4:1

Example 2. Simplify. \( \frac{9}{33} = \)

Solution: Both numbers 9 and 33 are divisible by 3, \( \Rightarrow 33 \div 3 = 11 \), 9 \( \div 3 = 3 \), Then: \( \frac{9}{33} = \frac{3}{11} \)

Example 3. There are 24 students in a class and 10 are girls. Write the ratio of girls to boys.

Solution: Subtract 10 from 24 to find the number of boys in the class. 24 \( - 10 = 14 \). There are 14 boys in the class. So, the ratio of girls to boys is 10:14. Now, simplify this ratio. Both 14 and 10 are divisible by 2. Then: 14 \( \div 2 = 7 \), and 10 \( \div 2 = 5 \). In the simplest form, this ratio is 5:7

Example 4. A recipe calls for butter and sugar in the ratio 3:4. If you're using 9 cups of butter, how many cups of sugar should you use?

Solution: Since you use 9 cups of butter, or 3 times as much, you need to multiply the amount of sugar by 3. Then: \( 4 \times 3 = 12 \). So, you need to use 12 cups of sugar. You can solve this using equivalent fractions: \( \frac{3}{4} = \frac{9}{12} \)
Proportional Ratios

- Two ratios are proportional if they represent the same relationship.
- A proportion means that two ratios are equal. It can be written in two ways: \( \frac{a}{b} = \frac{c}{d} \) and \( a : b = c : d \)
- The proportion \( \frac{a}{b} = \frac{c}{d} \) can be written as: \( a \times d = c \times b \)

Examples:

Example 1. Solve this proportion for \( x \). \( \frac{2}{5} = \frac{6}{x} \)

Solution: Use cross multiplication: \( \frac{2}{5} = \frac{6}{x} \Rightarrow 2 \times x = 6 \times 5 \Rightarrow 2x = 30 \)

Divide both sides by 2 to find \( x \): \( x = \frac{30}{2} \Rightarrow x = 15 \)

Example 2. If a box contains red and blue balls in ratio of 3:5 red to blue, how many red balls are there if 45 blue balls are in the box?

Solution: Write a proportion and solve. \( \frac{3}{5} = \frac{x}{45} \)

Use cross multiplication: \( 3 \times 45 = 5 \times x \Rightarrow 135 = 5x \)

Divide to find \( x \): \( x = \frac{135}{5} \Rightarrow x = 27 \). There are 27 red balls in the box.

Example 3. Solve this proportion for \( x \). \( \frac{4}{9} = \frac{16}{x} \)

Solution: Use cross multiplication: \( \frac{4}{9} = \frac{16}{x} \Rightarrow 4 \times x = 9 \times 16 \Rightarrow 4x = 144 \)

Divide to find \( x \): \( x = \frac{144}{4} \Rightarrow x = 36 \)

Example 4. Solve this proportion for \( x \). \( \frac{5}{7} = \frac{20}{x} \)

Solution: Use cross multiplication: \( \frac{5}{7} = \frac{20}{x} \Rightarrow 5 \times x = 7 \times 20 \Rightarrow 5x = 140 \)

Divide to find \( x \): \( x = \frac{140}{5} \Rightarrow x = 28 \)
Similarity and Ratios

- Two figures are similar if they have the same shape.

- Two or more figures are similar if the corresponding angles are equal, and the corresponding sides are in proportion.

Examples:

Example 1. The following triangles are similar. What is the value of the unknown side?

Solution: Find the corresponding sides and write a proportion.
\[
\frac{8}{16} = \frac{6}{x}
\]
Now, use the cross product to solve for \(x\):
\[
8 \times x = 16 \times 6 \rightarrow 8x = 96
\]
Divide both sides by 8. Then: 
\[
8x = 96 \rightarrow x = \frac{96}{8} \rightarrow x = 12
\]
The missing side is 12.

Example 3. Two rectangles are similar. The first is 5 feet wide and 15 feet long. The second is 10 feet wide. What is the length of the second rectangle?

Solution: Let’s put \(x\) for the length of the second rectangle. Since two rectangles are similar, their corresponding sides are in proportion. Write a proportion and solve for the missing number.
\[
\frac{5}{10} = \frac{15}{x} \rightarrow 5x = 10 \times 15 \rightarrow 5x = 150 \rightarrow x = \frac{150}{5} = 30
\]
The length of the second rectangle is 30 feet.
Chapter 4: Practices

Reduce each ratio.

1) $2:18 = \_:\_ 
7) $28:63 = \_:\_ 
2) $5:35 = \_:\_ 
8) $18:81 = \_:\_ 
3) $8:72 = \_:\_ 
9) $13:52 = \_:\_ 
4) $24:36 = \_:\_ 
10) $56:72 = \_:\_ 
5) $25:40 = \_:\_ 
11) $42:63 = \_:\_ 
6) $40:72 = \_:\_ 
12) $32:96 = \_:\_ 

Solve.

13) Bob has 16 red cards and 20 green cards. What is the ratio of Bob's red cards to his green cards? ________

14) In a party, 34 soft drinks are required for every 20 guests. If there are 260 guests, how many soft drinks are required? ________

15) Sara has 56 blue pens and 28 black pens. What is the ratio of Sara’s black pens to her blue pens? ________

16) In Jack’s class, 48 of the students are tall and 20 are short. In Michael’s class 28 students are tall and 12 students are short. Which class has a higher ratio of tall to short students? ________

17) The price of 6 apples at the Quick Market is $1.52. The price of 5 of the same apples at Walmart is $1.32. Which place is the better buy? ________

18) The bakers at a Bakery can make 180 bagels in 6 hours. How many bagels can they bake in 16 hours? What is that rate per hour? ________

19) You can buy 6 cans of green beans at a supermarket for $3.48. How much does it cost to buy 38 cans of green beans? ________
Solve each proportion.

20) $\frac{3}{2} = \frac{9}{x} \Rightarrow x = ____$

31) $\frac{4}{18} = \frac{2}{x} \Rightarrow x = ____$

21) $\frac{7}{2} = \frac{x}{4} \Rightarrow x = ____$

32) $\frac{6}{16} = \frac{3}{x} \Rightarrow x = ____$

22) $\frac{1}{3} = \frac{2}{x} \Rightarrow x = ____$

33) $\frac{2}{5} = \frac{x}{20} \Rightarrow x = ____$

23) $\frac{1}{4} = \frac{5}{x} \Rightarrow x = ____$

34) $\frac{28}{8} = \frac{x}{2} \Rightarrow x = ____$

24) $\frac{9}{6} = \frac{x}{2} \Rightarrow x = ____$

35) $\frac{3}{5} = \frac{x}{15} \Rightarrow x = ____$

25) $\frac{3}{6} = \frac{5}{x} \Rightarrow x = ____$

36) $\frac{2}{7} = \frac{x}{14} \Rightarrow x = ____$

26) $\frac{7}{x} = \frac{2}{6} \Rightarrow x = ____$

37) $\frac{x}{18} = \frac{3}{2} \Rightarrow x = ____$

27) $\frac{2}{x} = \frac{4}{10} \Rightarrow x = ____$

38) $\frac{x}{24} = \frac{2}{6} \Rightarrow x = ____$

28) $\frac{3}{2} = \frac{x}{8} \Rightarrow x = ____$

39) $\frac{5}{x} = \frac{4}{20} \Rightarrow x = ____$

29) $\frac{x}{6} = \frac{5}{3} \Rightarrow x = ____$

40) $\frac{10}{x} = \frac{20}{80} \Rightarrow x = ____$

30) $\frac{3}{9} = \frac{5}{x} \Rightarrow x = ____$

41) $\frac{90}{6} = \frac{x}{2} \Rightarrow x = ____$

Solve each problem.

42) Two rectangles are similar. The first is 8 feet wide and 32 feet long.
   The second is 12 feet wide. What is the length of the second rectangle?
   __________________

43) Two rectangles are similar. One is 4.6 meters by 7 meters. The longer
    side of the second rectangle is 28 meters. What is the other side of the
    second rectangle? __________________
Chapter 4: Answers

1) 1: 9  
2) 1: 7  
3) 1: 9  
4) 2: 3  
5) 5: 8  
6) 5: 9  
7) 4: 9  
8) 2: 9

9) 1: 4  
10) 7: 9  
11) 2: 3  
12) 1: 3  
13) 4: 5  
14) 442  
15) 1: 2

16) Jack’s class: \( \frac{48}{20} = \frac{12}{5} \)  
    Michael’s class: \( \frac{28}{12} = \frac{7}{3} \)  
    Jack’s class has a higher ratio of tall to short student: \( \frac{12}{5} > \frac{7}{3} \)

17) Quick market  
18) 480, 30 bagels per hour  
19) $22.04  
20) 6  
21) 14  
22) 6  
23) 20  
24) 3  
25) 10  
26) 21  
27) 5  
28) 12  
29) 10

30) 15  
31) 9  
32) 8  
33) 8  
34) 7  
35) 9  
36) 4  
37) 27  
38) 8  
39) 25  
40) 40  
41) 30  
42) 48 meters  
43) 18.4 meters
Math topics that you'll learn in this chapter:

- Percent Problems
- Percent of Increase and Decrease
- Discount, Tax and Tip
- Simple Interest
Percent Problems

- Percent is a ratio of a number and 100. It always has the same denominator, 100. The percent symbol is “%”.

- Percent means "per 100". So, 20% is 20/100.

- In each percent problem, we are looking for the base, or part or the percent.

- Use these equations to find each missing section in a percent problem:
  
  ❖ Base = Part ÷ Percent
  ❖ Part = Percent × Base
  ❖ Percent = Part ÷ Base

Examples:

Example 1. What is 20% of 40?

Solution: In this problem, we have percent (20%) and base (40) and we are looking for the “part”. Use this formula: part = percent × base.

Then: part = 20% × 40 = \(\frac{20}{100} \times 40 = 0.20 \times 40 = 8\). The answer: 20% of 40 is 8.

Example 2. 25 is what percent of 500?

Solution: In this problem, we are looking for the percent. Use this equation: Percent = Part ÷ Base → Percent = 25 ÷ 500 = 0.05 = 5%.

Then: 25 is 5 percent of 500.
Percent of Increase and Decrease

- Percent of change (increase or decrease) is a mathematical concept that represents the degree of change over time.
- To find the percentage of increase or decrease:
  1. New Number – Original Number
  2. The result ÷ Original Number × 100
- Or use this formula: Percent of change = \( \frac{\text{new number} - \text{original number}}{\text{original number}} \times 100 \)
- Note: If your answer is a negative number, then this is a percentage decrease. If it is positive, then this is a percentage increase.

Examples:

**Example 1.** The price of a shirt increases from $30 to $40. What is the percentage increase?

**Solution:** First, find the difference: 40 – 30 = 10

Then: 10 ÷ 30 × 100 = \( \frac{10}{30} \times 100 = 33.33 \). The percentage increase is 33.33. It means that the price of the shirt increased by 33.33%.

**Example 2.** The price of a table increased from $20 to $50. What is the percent of increase?

**Solution:** Use percentage formula:

\[
\text{Percent of change} = \frac{\text{new number} - \text{original number}}{\text{original number}} \times 100 = \frac{50-20}{20} \times 100 = \frac{30}{20} \times 100 = 1.5 \times 100 = 150.
\]

The percentage increase is 150. It means that the price of the table increased by 150%.
Discount, Tax and Tip

- To find the discount: Multiply the regular price by the rate of discount
- To find the selling price: Original price – discount
- To find tax: Multiply the tax rate to the taxable amount (income, property value, etc.)
- To find the tip, multiply the rate to the selling price.

Examples:

Example 1. With an 20% discount, Ella saved $50 on a dress. What was the original price of the dress?

Solution: let x be the original price of the dress. Then: 20 % of x = 50. Write an equation and solve for x: 0.20 × x = 50 → x = \frac{50}{0.20} = 250. The original price of the dress was $250.

Example 2. Sophia purchased a new computer for a price of $820 at the Apple Store. What is the total amount her credit card is charged if the sales tax is 5%?

Solution: The taxable amount is $820, and the tax rate is 5%. Then: Tax = 0.05 × 820 = 41
Final price = Selling price + Tax → final price = $820 + $41 = $861

Example 3. Nicole and her friends went out to eat at a restaurant. If their bill was $60.00 and they gave their server a 15% tip, how much did they pay altogether?

Solution: First, find the tip. To find the tip, multiply the rate to the bill amount. Tip = 60 × 0.15 = 9. The final price is: $60 + $9 = $69
Simple Interest

- Simple Interest: The charge for borrowing money or the return for lending it.
- Simple interest is calculated on the initial amount (principal).
- To solve a simple interest problem, use this formula:

\[ I = \text{principal} \times \text{rate} \times \text{time} \quad (I = p \times r \times t = \text{prt}) \]

Examples:

Example 1. Find simple interest for $200 investment at 5% for 3 years.

**Solution:** Use Interest formula:

\[ I = \text{prt} \quad (P = 200, \ r = 5\% = \frac{5}{100} = 0.05 \text{ and } t = 3) \]

Then: \[ I = 200 \times 0.05 \times 3 = 30 \]

Example 2. Find simple interest for $1,200 at 8% for 6 years.

**Solution:** Use Interest formula:

\[ I = \text{prt} \quad (P = 1,200, \ r = 8\% = \frac{8}{100} = 0.08 \text{ and } t = 6) \]

Then: \[ I = 1,200 \times 0.08 \times 6 = 576 \]

Example 3. Andy received a student loan to pay for his educational expenses this year. What is the interest on the loan if he borrowed $4,500 at 6% for 5 years?

**Solution:** Use Interest formula: \[ I = \text{prt} \quad (P = 4,500, \ r = 6\% = 0.06 \text{ and } t = 5) \]

Then: \[ I = 4,500 \times 0.06 \times 5 = 1,350 \]

Example 4. Bob is starting his own small business. He borrowed $20,000 from the bank at a 8% rate for 6 months. Find the interest Bob will pay on this loan.

**Solution:** Use Interest formula:

\[ I = \text{prt} \quad (P = 20,000, \ r = 8\% = 0.08 \text{ and } t = 0.5 \text{ (6 months is half year}) \]

Then: \[ I = 20,000 \times 0.08 \times 0.5 = 800 \]
Chapter 5: Practices

Solve each problem.

1) What is 15% of 60? ____ 7) 14 is what percent of 250? ____%
2) What is 55% of 800? ____ 8) 60 is what percent of 300? ____%
3) What is 22% of 120? ____ 9) 30 is 120 percent of what number? ___
4) What is 18% of 40? ____ 10) 120 is 20 percent of what number? ___
5) 90 is what percent of 200? ____% 11) 15 is 5 percent of what number? ___
6) 30 is what percent of 150? ____% 12) 22 is 20% of what number? ___

Solve each problem.

13) Bob got a raise, and his hourly wage increased from $15 to $21. What is the percent increase? _____ %
14) The price of a pair of shoes increases from $32 to $36. What is the percent increase? ___ %
15) At a Coffee Shop, the price of a cup of coffee increased from $1.35 to $1.62. What is the percent increase in the cost of the coffee? _____ %
16) A $45 shirt now selling for $36 is discounted by what percent? ____ %
17) Joe scored 30 out of 35 marks in Algebra, 20 out of 30 marks in science and 58 out of 70 marks in mathematics. In which subject his percentage of marks is best? _____
18) Emma purchased a computer for $420. The computer is regularly priced at $480. What was the percent discount Emma received on the computer? _____
19) A chemical solution contains 15% alcohol. If there is 54 ml of alcohol, what is the volume of the solution? _____
Find the selling price of each item.

20) Original price of a computer: $600
   Tax: 8%, Selling price: $_____

21) Original price of a laptop: $450
   Tax: 10%, Selling price: $_____

22) Nicolas hired a moving company. The company charged $500 for its services, and Nicolas gives the movers a 14% tip. How much does Nicolas tip the movers? $_____

23) Mason has lunch at a restaurant and the cost of his meal is $40. Mason wants to leave a 20% tip. What is Mason’s total bill, including tip? $_____

Determine the simple interest for the following loans.

24) $1,000 at 5% for 4 years.$____

25) $400 at 3% for 5 years.$____

26) $240 at 4% for 3 years.$____

27) $500 at 4.5% for 6 years.$____

Solve.

28) A new car, valued at $20,000, depreciates at 8% per year. What is the value of the car one year after purchase? $_________

29) Sara puts $7,000 into an investment yielding 3% annual simple interest; she left the money in for five years. How much interest does Sara get at the end of those five years? $_________
Chapter 5: Answers

1) 9
2) 440
3) 26.4
4) 7.2
5) 45%
6) 20%
7) 5.6%
8) 20%
9) 25
10) 600
11) 300
12) 110
13) 40%
14) 12.5%
15) 20%
16) 20%
17) Algebra
18) 12.5%
19) 360 ml
20) $648.00
21) $495.00
22) $70.00
23) $48.00
24) $200.00
25) $60.00
26) $28.80
27) $135.00
28) $18.400
29) $1,050
Math topics that you’ll learn in this chapter:

☑ Multiplication Property of Exponents
☑ Division Property of Exponents
☑ Powers of Products and Quotients
☑ Zero and Negative Exponents
☑ Negative Exponents and Negative Bases
☑ Scientific Notation
☑ Radicals
Multiplication Property of Exponents

- Exponents are shorthand for repeated multiplication of the same number by itself. For example, instead of $2 \times 2$, we can write $2^2$. For $3 \times 3 \times 3 \times 3$, we can write $3^4$.

- In algebra, a variable is a letter used to stand for a number. The most common letters are: $x, y, z, a, b, c, m, \text{ and } n$.

- Exponent’s rules: $x^a \times x^b = x^{a+b}$, $\frac{x^a}{x^b} = x^{a-b}\n$

$$(x^a)^b = x^{a\times b}$$

$$(xy)^a = x^a \times y^a$$

$$\left(\frac{a}{b}\right)^c = \frac{a^c}{b^c}\n$$

Examples:

**Example 1.** Multiply. $2x^2 \times 3x^4$

**Solution:** Use Exponent’s rules: $x^a \times x^b = x^{a+b} \rightarrow x^2 \times x^4 = x^{2+4} = x^6$

Then: $2x^2 \times 3x^4 = 6x^6$

**Example 2.** Simplify. $(x^4y^2)^2$

**Solution:** Use Exponent’s rules: $(x^a)^b = x^{a\times b}$.

Then: $(x^4y^2)^2 = x^{4\times 2}y^{2\times 2} = x^8y^4$

**Example 3.** Multiply. $5x^8 \times 6x^5$

**Solution:** Use Exponent’s rules: $x^a \times x^b = x^{a+b} \rightarrow x^8 \times x^5 = x^{8+5} = x^{13}$

Then: $5x^8 \times 6x^5 = 30x^{13}$

**Example 4.** Simplify. $(x^4y^7)^3$

**Solution:** Use Exponent’s rules: $(x^a)^b = x^{a\times b}$.

Then: $(x^4y^7)^3 = x^{4\times 3}y^{7\times 3} = x^{12}y^{21}$
Division Property of Exponents

- Exponents are shorthand for repeated multiplication of the same number by itself. For example, instead of $3 \times 3$, we can write $3^2$. For $2 \times 2 \times 2$, we can write $2^3$

- For division of exponents use following formulas:

$$\frac{x^a}{x^b} = x^{a-b}, x \neq 0, \quad \frac{x^a}{x^{b-a}} = \frac{1}{x^{b-a}}, x \neq 0, \quad \frac{1}{x^b} = x^{-b}$$

Examples:

Example 1. Simplify. $\frac{16x^3y}{2xy^2} =$

Solution: First, cancel the common factor: $2 \to \frac{16x^3y}{2xy^2} = \frac{8x^3y}{xy^2}$

Use Exponent’s rules: $\frac{x^a}{x^b} = x^{a-b} \to \frac{x^3}{x} = x^{3-1} = x^2$ and $\frac{y}{y^2} = \frac{1}{y^{2-1}} = \frac{1}{y}$

Then: $\frac{16x^3y}{2xy^2} = \frac{8x^2}{y}$

Example 2. Simplify. $\frac{24x^8}{3x^6} =$

Solution: Use Exponent’s rules: $\frac{x^a}{x^b} = x^{b-a} \to \frac{x^8}{x^6} = x^{8-6} = x^2$

Then: $\frac{24x^8}{3x^6} = 8x^2$

Example 3. Simplify. $\frac{7x^4y^2}{28x^3y} =$

Solution: First, cancel the common factor: $7 \to \frac{7x^4y^2}{4x^3y}$

Use Exponent’s rules: $\frac{x^a}{x^b} = x^{a-b} \to \frac{x^4}{x^3} = x^{4-3} = x$ and $\frac{y^2}{y} = y$

Then: $\frac{7x^4y^2}{28x^3y} = \frac{xy}{4}$
Powers of Products and Quotients

- Exponents are shorthand for repeated multiplication of the same number by itself. For example, instead of $2 \times 2 \times 2$, we can write $2^3$. For $3 \times 3 \times 3 \times 3$, we can write $3^4$

- For any nonzero numbers $a$ and $b$ and any integer $x$, $(ab)^x = a^x \times b^x$

And $(\frac{a}{b})^x = \frac{a^x}{b^x}$

Examples:

Example 1. Simplify. $(3x^3y^2)^2$

Solution: Use Exponent’s rules: $(x^a)^b = x^{a\times b}$

$(3x^3y^2)^2 = (3)^2(x^3)^2(y^2)^2 = 9x^{3\times2}y^{2\times2} = 9x^6y^4$

Example 2. Simplify. $\left(\frac{2x^3}{3x^2}\right)^2$

Solution: First, cancel the common factor: $x \rightarrow \left(\frac{2x^3}{3x^2}\right) = \left(\frac{2x}{3}\right)^2$

Use Exponent’s rules: $(\frac{a}{b})^x = \frac{a^x}{b^x}$. Then: $\left(\frac{2x}{3}\right)^2 = \frac{(2x)^2}{(3)^2} = \frac{4x^2}{9}$

Example 3. Simplify. $(-4x^3y^5)^2$

Solution: Use Exponent’s rules: $(x^a)^b = x^{a\times b}$

$(-4x^3y^5)^2 = (-4)^2(x^3)^2(y^5)^2 = 16x^{3\times2}y^{5\times2} = 16x^6y^{10}$

Example 4. Simplify. $\left(\frac{5x}{4x^2}\right)^2$

Solution: First, cancel the common factor: $x \rightarrow \left(\frac{5x}{4x^2}\right)^2 = \left(\frac{5}{4x}\right)^2$

Use Exponent’s rules: $(\frac{a}{b})^x = \frac{a^x}{b^x}$. Then: $\left(\frac{5}{4x}\right)^2 = \frac{5^2}{(4x)^2} = \frac{25}{16x^2}$
Zero and Negative Exponents

- Zero-Exponent Rule: \(a^0 = 1\), this means that anything raised to the zero power is 1. For example: \((5xy)^0 = 1\)

- A negative exponent simply means that the base is on the wrong side of the fraction line, so you need to flip the base to the other side. For instance, "x\(^{-2}\)" (pronounced as "ecks to the minus two") just means "x\(^2\)" but underneath, as in \(\frac{1}{x^2}\).

Examples:

Example 1. Evaluate. \(\left(\frac{4}{5}\right)^{-2}\) =

\[\text{Solution:}\] Use negative exponent’s rule: \(\left(\frac{x^a}{x^b}\right)^{-2} = \left(\frac{x^b}{x^a}\right)^{-2}\)

\(\rightarrow\) \(\left(\frac{4}{5}\right)^{-2} = \left(\frac{5}{4}\right)^2 = \frac{25}{16}\)

Example 2. Evaluate. \(\left(\frac{3}{2}\right)^{-3}\) =

\[\text{Solution:}\] Use negative exponent’s rule: \(\left(\frac{x^a}{x^b}\right)^{-3} = \left(\frac{x^b}{x^a}\right)^{-3}\)

\(\rightarrow\) \(\left(\frac{3}{2}\right)^{-3} = \left(\frac{2}{3}\right)^3 = \frac{8}{27}\)

Example 3. Evaluate. \(\left(\frac{a}{b}\right)^0\) =

\[\text{Solution:}\] Use zero-exponent Rule: \(a^0 = 1\)

\(\rightarrow\) \(\left(\frac{a}{b}\right)^0 = 1\)

Example 4. Evaluate. \(\left(\frac{4}{7}\right)^{-1}\) =

\[\text{Solution:}\] Use negative exponent’s rule: \(\left(\frac{x^a}{x^b}\right)^{-1} = \left(\frac{x^b}{x^a}\right)^{-1}\)

\(\rightarrow\) \(\left(\frac{4}{7}\right)^{-1} = \left(\frac{7}{4}\right)^1 = \frac{7}{4}\)
Negative Exponents and Negative Bases

- A negative exponent is the reciprocal of that number with a positive exponent. \(3^{-2} = \frac{1}{3^2}\)

- To simplify a negative exponent, make the power positive!

- The parenthesis is important! \(-5^{-2}\) is not the same as \((-5)^{-2}\)

\[-5^{-2} = -\frac{1}{5^2}\] and \((-5)^{-2} = +\frac{1}{5^2}\)

Examples:

Example 1. Simplify. \((\frac{2a}{3c})^{-2}\) =

**Solution**: Use negative exponent’s rule: \((x^{a})^{-2} = (\frac{x^b}{x^a})\) → \((\frac{2a}{3c})^{-2} = (\frac{3c}{2a})^2\)

Now use exponent’s rule: \((\frac{a}{b})^c = a^c \rightarrow (\frac{3c}{2a})^2 = \frac{3^2c^2}{2^2a^2}\)

Then: \(\frac{3^2c^2}{2^2a^2} = \frac{9c^2}{4a^2}\)

Example 2. Simplify. \((\frac{x}{4y})^{-3}\) =

**Solution**: Use negative exponent’s rule: \((\frac{x^a}{y^b})^{-3} = (\frac{x^b}{y^a})^3\) → \((\frac{x}{4y})^{-3} = (\frac{4y}{x})^3\)

Now use exponent’s rule: \((\frac{a}{b})^c = a^c \rightarrow (\frac{4y}{x})^3 = \frac{4^3y^3}{x^3} = \frac{64y^3}{x^3}\)

Example 3. Simplify. \((\frac{5a}{2c})^{-2}\) =

**Solution**: Use negative exponent’s rule: \((\frac{x^a}{y^b})^{-2} = (\frac{x^b}{y^a})^2\) → \((\frac{5a}{2c})^{-2} = (\frac{2c}{5a})^2\)

Now use exponent’s rule: \((\frac{a}{b})^c = a^c \rightarrow (\frac{5a}{2c})^2 = \frac{2^2c^2}{5^2a^2}\)

Then: \(\frac{2^2c^2}{5^2a^2} = \frac{4c^2}{25a^2}\)
Scientific Notation

- Scientific notation is used to write very big or very small numbers in decimal form.
- In scientific notation, all numbers are written in the form of: $m \times 10^n$, where $m$ is greater than 1 and less than 10.
- To convert a number from scientific notation to standard form, move the decimal point to the left (if the exponent of ten is a negative number), or to the right (if the exponent is positive).

Examples:

Example 1. Write 0.00024 in scientific notation.

Solution: First, move the decimal point to the right so you have a number between 1 and 10. That number is 2.4. Now, determine how many places the decimal moved in step 1 by the power of 10. We moved the decimal point 4 digits to the right. Then: $10^{-4} \rightarrow$ When the decimal moved to the right, the exponent is negative. Then: $0.00024 = 2.4 \times 10^{-4}$

Example 2. Write $3.8 \times 10^{-5}$ in standard notation.

Solution: $10^{-5} \rightarrow$ When the decimal moved to the right, the exponent is negative. Then: $3.8 \times 10^{-5} = 0.000038$

Example 3. Write 0.00031 in scientific notation.

Solution: First, move the decimal point to the right so you have a number between 1 and 10. Then: $m = 3.1$ , Now, determine how many places the decimal moved in step 1 by the power of 10. $10^{-4} \rightarrow$ Then: $0.00031 = 3.1 \times 10^{-4}$

Example 4. Write $6.2 \times 10^{5}$ in standard notation.

Solution: $10^{5} \rightarrow$ The exponent is positive 5. Then, move the decimal point to the right five digits. (remember $6.2 = 6.20000$), Then: $6.2 \times 10^{5} = 620000$
Radicals

- If \( n \) is a positive integer and \( x \) is a real number, then: \( \sqrt[n]{x} = x^{\frac{1}{n}} \),

\[
\sqrt[n]{xy} = \sqrt[n]{x} \times \sqrt[n]{y}, \quad \sqrt[n]{\frac{x}{y}} = \frac{\sqrt[n]{x}}{\sqrt[n]{y}}, \quad \text{and} \quad \sqrt[n]{x} \times \sqrt[n]{y} = \sqrt[n]{xy}
\]

- A square root of \( x \) is a number \( r \) whose square is: \( r^2 = x \) (\( r \) is a square root of \( x \))

- To add and subtract radicals, we need to have the same values under the radical. For example: \( \sqrt{3} + \sqrt{3} = 2\sqrt{3}, \quad 3\sqrt{5} - \sqrt{5} = 2\sqrt{5} \)

Examples:

Example 1. Find the square root of \( \sqrt{121} \).

Solution: First, factor the number: \( 121 = 11^2 \), Then: \( \sqrt{121} = \sqrt{11^2} \),

Now use radical rule: \( \sqrt[n]{a^n} = a \). Then: \( \sqrt{121} = \sqrt{11^2} = 11 \)

Example 2. Evaluate. \( \sqrt{4} \times \sqrt{16} = \)

Solution: Find the values of \( \sqrt{4} \) and \( \sqrt{16} \). Then: \( \sqrt{4} \times \sqrt{16} = 2 \times 4 = 8 \)

Example 3. Solve. \( 5\sqrt{2} + 9\sqrt{2} \).

Solution: Since we have the same values under the radical, we can add these two radicals: \( 5\sqrt{2} + 9\sqrt{2} = 14\sqrt{2} \)

Example 4. Evaluate. \( \sqrt{2} \times \sqrt{50} = \)

Solution: Use this radical rule: \( \sqrt[n]{x} \times \sqrt[n]{y} = \sqrt[n]{xy} \rightarrow \sqrt{2} \times \sqrt{50} = \sqrt{100} \)

The square root of 100 is 10. Then: \( \sqrt{2} \times \sqrt{50} = \sqrt{100} = 10 \)
Chapter 6: Exponents and Variables

Find the products.

1) \( x^2 \times 4xy^2 = \)

2) \( 3x^2y \times 5x^3y^2 = \)

3) \( 6x^4y^2 \times x^2y^3 = \)

4) \( 7xy^3 \times 2x^2y = \)

5) \( -5x^5y^5 \times x^3y^2 = \)

6) \( -8x^3y^2 \times 3x^3y^2 = \)

7) \( -6x^2y^6 \times 5x^4y^2 = \)

8) \( -3x^3y^3 \times 2x^3y^2 = \)

9) \( -6x^5y^3 \times 4x^4y^3 = \)

10) \( -2x^4y^3 \times 5x^6y^2 = \)

11) \( -7y^6 \times 3x^6y^3 = \)

12) \( -9x^4 \times 2x^4y^2 = \)

Simplify.

13) \( \frac{3x \times 5^4}{3^5 \times 5} = \)

14) \( \frac{3^3 \times 3^2}{7^3 \times 7} = \)

15) \( \frac{15x^5}{5x^3} = \)

16) \( \frac{16x^3}{4x^5} = \)

17) \( \frac{72y^2}{8x^3y^6} = \)

18) \( \frac{10x^3y^4}{50x^2y^3} = \)

19) \( \frac{13y^2}{52x^4y^4} = \)

20) \( \frac{50xy^2}{200x^3y^4} = \)

21) \( \frac{48x^2}{56x^2y^2} = \)

22) \( \frac{81y^6x}{54x^4y^3} = \)

Solve.

23) \( (x^3y^3)^2 = \)

24) \( (3x^3y^4)^3 = \)

25) \( (4x \times 6xy^3)^2 = \)

26) \( (5x \times 2y^3)^3 = \)

27) \( \left( \frac{9x^2}{y^3} \right)^2 = \)

28) \( \left( \frac{3y}{18y^2} \right)^2 = \)

29) \( \left( \frac{3x^2y^3}{24x^4y^2} \right)^3 = \)

30) \( \left( \frac{26x^5y^3}{52x^3y^5} \right)^2 = \)

31) \( \left( \frac{18x^7y^4}{72x^5y^2} \right)^2 = \)

32) \( \left( \frac{12x^6y^4}{48x^5y^3} \right)^2 = \)
Evaluate each expression. (Zero and Negative Exponents)

33) \( \left( \frac{1}{4} \right)^{-2} = \)

34) \( \left( \frac{1}{3} \right)^{-2} = \)

35) \( \left( \frac{1}{7} \right)^{-3} = \)

36) \( \left( \frac{2}{5} \right)^{-3} = \)

37) \( \left( \frac{2}{3} \right)^{-3} = \)

38) \( \left( \frac{3}{5} \right)^{-4} = \)

Write each expression with positive exponents.

39) \( x^{-7} = \)

40) \( 3y^{-5} = \)

41) \( 15y^{-3} = \)

42) \( -20x^{-4} = \)

43) \( 12a^{-3}b^5 = \)

44) \( 25a^3b^{-4}c^{-3} = \)

45) \( -4x^5y^{-3}z^{-6} = \)

46) \( \frac{18y}{x^3y^{-2}} = \)

47) \( \frac{20a^{-2}b}{-12c^{-4}} = \)

Write each number in scientific notation.

48) \( 0.00412 = \)

49) \( 0.000053 = \)

50) \( 66,000 = \)

51) \( 72,000,000 = \)

Evaluate.

52) \( \sqrt{8} \times \sqrt{8} = \)

53) \( \sqrt{36} - \sqrt{9} = \)

54) \( \sqrt{81} + \sqrt{16} = \)

55) \( \sqrt{4} \times \sqrt{25} = \)

56) \( \sqrt{2} \times \sqrt{32} = \)

57) \( 4\sqrt{3} + 5\sqrt{3} = \)
Chapter 6: Answers

1) $4x^3y^2$
2) $15x^5y^3$
3) $6x^6y^5$
4) $14x^3y^4$
5) $-5x^8y^7$
6) $-24x^6y^4$
7) $-30x^6y^8$
8) $-6x^6y^5$
9) $-24x^9y^6$
10) $-10x^{10}y^5$
11) $-21x^6y^9$
12) $-18x^8y^2$
13) $\frac{1}{125}$
14) $\frac{243}{343}$
15) $3x^2$
16) $\frac{4}{x^2}$
17) $\frac{9}{x^2y^4}$
18) $\frac{xy}{5}$
19) $\frac{1}{4x^4y^2}$
20) $\frac{1}{4x^2y}$
21) $\frac{6}{7y^2}$
22) $\frac{3y^5}{2x^3}$
23) $x^6y^6$
24) $27x^9y^{12}$
25) $576x^4y^6$
26) $1,000x^3y^9$
27) $\frac{81}{x^4}$
28) $\frac{1}{36y^2}$
29) $\frac{y^3}{512x^6}$
30) $\frac{x^4}{4y^4}$
31) $\frac{x^4y^4}{16}$
32) $\frac{x^2y^2}{16}$
33) 16
34) 9
35) 343
36) $\frac{125}{8}$
37) $\frac{27}{8}$
38) $\frac{625}{81}$
39) $\frac{1}{x^7}$
40) $\frac{3}{y^5}$
41) $\frac{15}{y^3}$
42) $-\frac{20}{x^4}$
43) $\frac{12b^5}{a^3}$
44) $\frac{25a^3}{b^4c^3}$
45) $-\frac{4x^5}{y^3z^6}$
46) $\frac{18y^3}{x^3}$
47) $-\frac{5bc^4}{3a^2}$
48) $4.12 \times 10^{-3}$
49) $5.3 \times 10^{-5}$
50) $6.6 \times 10^4$
51) $7.2 \times 10^7$
52) 8
53) 3
54) 13
55) 10
56) 8
57) $9\sqrt{3}$
Math topics that you’ll learn in this chapter:

- ✔ Simplifying Variable Expressions
- ✔ Simplifying Polynomial Expressions
- ✔ The Distributive Property
- ✔ Evaluating One Variable
- ✔ Evaluating Two Variables
Simplifying Variable Expressions

- In algebra, a variable is a letter used to stand for a number. The most common letters are $x, y, z, a, b, c, m, \text{and } n$.
- An algebraic expression is an expression that contains integers, variables, and math operations such as addition, subtraction, multiplication, division, etc.
- In an expression, we can combine “like” terms. (values with same variable and same power)

Examples:

Example 1. Simplify. $(4x + 2x + 4) =$

Solution: In this expression, there are three terms: $4x, 2x$, and $4$. Two terms are “like terms”: $4x$ and $2x$. Combine like terms. $4x + 2x = 6x$. Then: $(4x + 2x + 4) = 6x + 4$ (remember you cannot combine variables and numbers.)

Example 2. Simplify. $-2x^2 - 5x + 4x^2 - 9 =$

Solution: Combine “like” terms: $-2x^2 + 4x^2 = 2x^2$.
Then: $-2x^2 - 5x + 4x^2 - 9 = 2x^2 - 5x - 9$.

Example 3. Simplify. $(-8 + 6x^2 + 3x^2 + 9x) =$

Solution: Combine like terms. Then:
$(-8 + 6x^2 + 3x^2 + 9x) = 9x^2 + 9x - 8$

Example 4. Simplify. $-10x + 6x^2 - 3x + 9x^2 =$

Solution: Combine “like” terms: $-10x - 3x = -13x$, and $6x^2 + 9x^2 = 15x^2$
Then: $-10x + 6x^2 - 3x + 9x^2 = -13x + 15x^2$. Write in standard form (biggest powers first): $-13x + 15x^2 = 15x^2 - 13x$
Simplifying Polynomial Expressions

- In mathematics, a polynomial is an expression consisting of variables and coefficients that involves only the operations of addition, subtraction, multiplication, and non-negative integer exponents of variables. $P(x) = a_n x^n + a_{n-1} x^{n-1} + ... + a_2 x^2 + a_1 x + a_0$

- Polynomials must always be simplified as much as possible. It means you must add together any like terms. (values with same variable and same power)

Examples:

Example 1. Simplify this Polynomial Expressions. $3x^2 - 6x^3 - 2x^3 + 4x^4$

Solution: Combine “like” terms: $-6x^3 - 2x^3 = -8x^3$

Then: $3x^2 - 6x^3 - 2x^3 + 4x^4 = 3x^2 - 8x^3 + 4x^4$

Now, write the expression in standard form: $4x^4 - 8x^3 + 3x^2$

Example 2. Simplify this expression. $(-5x^2 + 2x^3) - (3x^3 - 6x^2) =$

Solution: First, use distributive property: $→$ multiply $(-)$ into $(3x^3 - 6x^2)$

$(-5x^2 + 2x^3) - (3x^3 - 6x^2) = -5x^2 + 2x^3 - 3x^3 + 6x^2$

Then combine “like” terms: $-5x^2 + 2x^3 - 3x^3 + 6x^2 = x^2 - x^3$

And write in standard form: $x^2 - x^3 = -x^3 + x^2$

Example 3. Simplify. $3x^3 - 9x^4 - 8x^2 + 12x^4 =$

Solution: Combine “like” terms:

$-9x^4 + 12x^4 = 3x^4$

Then: $3x^3 - 9x^4 - 8x^2 + 12x^4 = 3x^3 + 3x^4 - 8x^2$

And write in standard form: $3x^3 + 3x^4 - 8x^2 = 3x^4 + 3x^3 - 8x^2$
The Distributive Property

- The distributive property (or the distributive property of multiplication over addition and subtraction) simplifies and solves expressions in the form of: \(a(b + c)\) or \(a(b - c)\)
- The distributive property is multiplying a term outside the parentheses by the terms inside.
- Distributive Property rule: \(a(b + c) = ab + ac\)

Examples:

Example 1. Simply using the distributive property. \((-2)(x + 3)\)

Solution: Use Distributive Property rule: \(a(b + c) = ab + ac\)
\((-2)(x + 3) = (-2 \times x) + (-2) \times (3) = -2x - 6\)

Example 2. Simply. \((-5)(-2x - 6)\)

Solution: Use Distributive Property rule: \(a(b + c) = ab + ac\)
\((-5)(-2x - 6) = (-5 \times -2x) + (-5) \times (-6) = 10x + 30\)

Example 3. Simply. \((7)(2x - 8) - 12x\)

Solution: First, simplify \((7)(2x - 8)\) using the distributive property.
Then: \((7)(2x - 8) = 14x - 56\)
Now combine like terms: \((7)(2x - 8) - 12x = 14x - 56 - 12x\)
In this expression, 14x and -12x are “like terms” and we can combine them.
14x - 12x = 2x. Then: 14x - 56 - 12x = 2x - 56
Evaluating One Variable

- To evaluate one variable expression, find the variable and substitute a number for that variable.
- Perform the arithmetic operations.

Examples:

Example 1. Calculate this expression for \( x = 2 \). \( 8 + 2x \)

Solution: First, substitute 2 for \( x \)
Then: \( 8 + 2x = 8 + 2(2) \)
Now, use order of operation to find the answer: \( 8 + 2(2) = 8 + 4 = 12 \)

Example 2. Evaluate this expression for \( x = -1 \). \( 4x - 8 \)

Solution: First, substitute \(-1\) for \( x \),
Then: \( 4x - 8 = 4(-1) - 8 \)
Now, use order of operation to find the answer: \( 4(-1) - 8 = -4 - 8 = -12 \)

Example 3. Find the value of this expression when \( x = 4 \). \( 16 - 5x \)

Solution: First, substitute 4 for \( x \),
Then: \( 16 - 5x = 16 - 5(4) = 16 - 20 = -4 \)

Example 4. Solve this expression for \( x = -3 \). \( 15 + 7x \)

Solution: Substitute \(-3\) for \( x \),
Then: \( 15 + 7x = 15 + 7(-3) = 15 - 21 = -6 \)
Evaluating Two Variables

- To evaluate an algebraic expression, substitute a number for each variable.
- Perform the arithmetic operations to find the value of the expression.

Examples:

Example 1. Calculate this expression for $a = 2$ and $b = -1$. $4a - 3b$

Solution: First, substitute 2 for $a$, and $-1$ for $b$.
Then: $4a - 3b = 4(2) - 3(-1)$
Now, use order of operation to find the answer: $4(2) - 3(-1) = 8 + 3 = 11$

Example 2. Evaluate this expression for $x = -2$ and $y = 2$. $3x + 6y$

Solution: Substitute $-2$ for $x$, and 2 for $y$.
Then: $3x + 6y = 3(-2) + 6(2) = -6 + 12 = 6$

Example 3. Find the value of this expression $2(6a - 5b)$ when $a = -1$ and $b = 4$.

Solution: Substitute $-1$ for $a$, and 4 for $b$.
Then: $2(6a - 5b) = 12a - 10b = 12(-1) - 10(4) = -12 - 40 = -52$

Example 4. Solve this expression. $-7x - 2y$, $x = 4$, $y = -3$

Solution: Substitute 4 for $x$, and $-3$ for $y$ and simplify.
Then: $-7x - 2y = -7(4) - 2(-3) = -28 + 6 = -22$
Chapter 7: Practices

**Simplify each expression.**

1) $(3 + 4x - 1) =
2) (-5 - 2x + 7) =
3) $(12x - 5x - 4) =
4) $(-16x + 24x - 9) =
5) $(6x + 5 - 15x) =
6) $2 + 5x - 8x - 6 =
7) $5x + 10 - 3x - 22 =

8) $-5 - 3x^2 - 6 + 4x =
9) $-6 + 9x^2 - 3 + x =
10) $5x^2 + 3x - 10x - 3 =
11) $4x^2 - 2x - 6x + 5 - 8 =
12) $3x^2 - 5x - 7x + 2 - 4 =
13) $9x^2 - x - 5x + 3 - 9 =
14) $2x^2 - 7x - 3x^2 + 4x + 6 =

**Simplify each polynomial.**

15) $5x^2 + 3x^3 - 9x^2 + 2x =
16) $8x^4 + 2x^5 - 7x^4 + 3x^2 =
17) $15x^3 + 11x - 5x^2 - 9x^3 =
18) $(7x^3 - 3x^2) + (5x^2 - 13x) =
19) $(12x^4 + 6x^3) + (x^3 - 5x^4) =
20) $(15x^5 - 8x^3) - (4x^3 + x^2) =
21) $(14x^4 + 7x^3) - (x^3 - 24) =
22) $(20x^4 + 6x^3) - (-x^3 - 2x^4) =
23) $(x^2 + 9x^3) + (-22x^2 + 6x^3) =
24) $(4x^4 - 2x^3) + (-5x^3 - 8x^4) =$
Use the distributive property to simply each expression.

25) \(2(6 + x) = \) \[\text{_____}\]  
26) \(5(3 - 2x) = \) \[\text{_____}\]  
27) \(7(1 - 5x) = \) \[\text{_____}\]  
28) \((3 - 4x)7 = \) \[\text{_____}\]  
29) \(6(2 - 3x) = \) \[\text{_____}\]  
30) \((-1)(-9 + x) = \) \[\text{_____}\]  
31) \((-6)(3x - 2) = \) \[\text{_____}\]  
32) \((-x + 12)(-4) = \) \[\text{_____}\]  
33) \((-2)(1 - 6x) = \) \[\text{_____}\]  
34) \((-5x - 3)(-8) = \) \[\text{_____}\]  

Evaluate each expression using the value given.

35) \(x = 4 \rightarrow 10 - x = \) \[\text{___}\]  
36) \(x = 6 \rightarrow x + 8 = \) \[\text{___}\]  
37) \(x = 3 \rightarrow 2x - 6 = \) \[\text{___}\]  
38) \(x = 2 \rightarrow 10 - 4x = \) \[\text{___}\]  
39) \(x = 7 \rightarrow 8x - 3 = \) \[\text{___}\]  
40) \(x = 9 \rightarrow 20 - 2x = \) \[\text{___}\]  
41) \(x = 5 \rightarrow 10x - 30 = \) \[\text{___}\]  
42) \(x = -6 \rightarrow 5 - x = \) \[\text{___}\]  
43) \(x = -3 \rightarrow 22 - 3x = \) \[\text{___}\]  
44) \(x = -7 \rightarrow 10 - 9x = \) \[\text{___}\]  
45) \(x = -10 \rightarrow 40 - 3x = \) \[\text{___}\]  
46) \(x = -2 \rightarrow 20x - 5 = \) \[\text{___}\]  
47) \(x = -5 \rightarrow -10x - 8 = \) \[\text{___}\]  
48) \(x = -4 \rightarrow -1 - 4x = \) \[\text{___}\]  

Evaluate each expression using the values given.

49) \(x = 2, y = 1 \rightarrow 2x + 7y = \) \[\text{_____}\]  
50) \(a = 3, b = 5 \rightarrow 3a - 5b = \) \[\text{_____}\]  
51) \(x = 6, y = 2 \rightarrow 3x - 2y + 8 = \) \[\text{_____}\]  
52) \(a = -2, b = 3 \rightarrow -5a + 2b + 6 = \) \[\text{_____}\]  
53) \(x = -4, y = -3 \rightarrow -4x + 10 - 8y = \) \[\text{_____}\]
Chapter 7: Answers

1) \(4x + 2\)  
2) \(-2x + 2\)  
3) \(7x - 4\)  
4) \(8x - 9\)  
5) \(-9x + 5\)  
6) \(-3x - 4\)  
7) \(2x - 12\)  
8) \(-3x^2 + 4x - 11\)  
9) \(9x^2 + x - 9\)  
10) \(5x^2 - 7x - 3\)  
11) \(4x^2 - 8x - 3\)  
12) \(3x^2 - 12x - 2\)  
13) \(9x^2 - 6x - 6\)  
14) \(-x^2 - 3x + 6\)  
15) \(3x^3 - 4x^2 + 2x\)  
16) \(2x^5 + x^4 + 3x^2\)  
17) \(6x^3 - 5x^2 + 11x\)  
18) \(7x^3 + 2x^2 - 13x\)  
19) \(7x^4 + 7x^3\)  
20) \(15x^5 - 12x^3 - x^2\)  
21) \(14x^4 + 6x^3 + 24\)  
22) \(22x^4 + 7x^3\)  
23) \(15x^3 - 21x^2\)  
24) \(-4x^4 - 7x^3\)  
25) \(2x + 12\)  
26) \(-10x + 15\)  
27) \(-35x + 7\)  
28) \(-28x + 21\)  
29) \(-18x + 12\)  
30) \(-x + 9\)  
31) \(-18x + 12\)  
32) \(4x - 48\)  
33) \(12x - 2\)  
34) \(40x + 24\)  
35) \(6\)  
36) \(14\)  
37) \(0\)  
38) \(2\)  
39) \(53\)  
40) \(2\)  
41) \(20\)  
42) \(11\)  
43) \(31\)  
44) \(73\)  
45) \(70\)  
46) \(-45\)  
47) \(42\)  
48) \(15\)  
49) \(11\)  
50) \(-16\)  
51) \(22\)  
52) \(22\)  
53) \(50\)
Math topics that you’ll learn in this chapter:

- One-Step Equations
- Multi-Step Equations
- System of Equations
- Graphing Single–Variable Inequalities
- One-Step Inequalities
- Multi-Step Inequalities
One–Step Equations

- The values of two expressions on both sides of an equation are equal. Example: \( ax = b \). In this equation, \( ax \) is equal to \( b \).

- Solving an equation means finding the value of the variable.

- You only need to perform one Math operation to solve the one-step equations.

- To solve a one-step equation, find the inverse (opposite) operation is being performed.

- The inverse operations are:
  - Addition and subtraction
  - Multiplication and division

Examples:

Example 1. Solve this equation for \( x \). \( 4x = 16, x = ? \)

Solution: Here, the operation is multiplication (variable \( x \) is multiplied by 4) and its inverse operation is division. To solve this equation, divide both sides of equation by 4: \( 4x = 16 \rightarrow \frac{4x}{4} = \frac{16}{4} \rightarrow x = 4 \)

Example 2. Solve this equation. \( x + 8 = 0, x = ? \)

Solution: In this equation 8 is added to the variable \( x \). The inverse operation of addition is subtraction. To solve this equation, subtract 8 from both sides of the equation: \( x + 8 - 8 = 0 - 8 \). Then: \( \rightarrow x = -8 \)

Example 3. Solve this equation for \( x \). \( x + 12 = 0 \)

Solution: Here, the operation is addition and its inverse operation is subtraction. To solve this equation, subtract 12 from both sides of the equation: \( x + 12 - 12 = 0 - 12 \rightarrow x = -12 \)
Multi–Step Equations

- To solve a multi-step equation, combine “like” terms on one side.
- Bring variables to one side by adding or subtracting.
- Simplify using the inverse of addition or subtraction.
- Simplify further by using the inverse of multiplication or division.
- Check your solution by plugging the value of the variable into the original equation.

Examples:

Example 1. Solve this equation for \(x\). \(4x + 8 = 20 - 2x\)

Solution: First, bring variables to one side by adding \(2x\) to both sides. Then:

\[4x + 8 + 2x = 20 - 2x + 2x \rightarrow 4x + 8 + 2x = 20.\]

Simplify: \(6x + 8 = 20\) Now, subtract 8 from both sides of the equation:

\[6x + 8 - 8 = 20 - 8 \rightarrow 6x = 12 \rightarrow \text{Divide both sides by 6:} \]

\[6x = 12 \rightarrow \frac{6x}{6} = \frac{12}{6} \rightarrow x = 2\]

Let’s check this solution by substituting the value of 2 for \(x\) in the original equation:

\(x = 2 \rightarrow 4x + 8 = 20 - 2x \rightarrow 4(2) + 8 = 20 - 2(2) \rightarrow 16 = 16\)

The answer \(x = 2\) is correct.

Example 2. Solve this equation for \(x\). \(-5x + 4 = 24\)

Solution: Subtract 4 from both sides of the equation.

\[-5x + 4 = 24 \rightarrow -5x + 4 - 4 = 24 - 4 \rightarrow -5x = 20\]

Divide both sides by \(-5\), then: \(-5x = 20 \rightarrow \frac{-5x}{-5} = \frac{20}{-5} \rightarrow x = -4\)

Now, check the solution:

\(x = -4 \rightarrow -5x + 4 = 24 \rightarrow -5(-4) + 4 = 24 \rightarrow 24 = 24\)

The answer \(x = -4\) is correct.
System of Equations

- A system of equations contains two equations and two variables. For example, consider the system of equations: \( x - y = 1, x + y = 5 \)

- The easiest way to solve a system of equations is using the elimination method. The elimination method uses the addition property of equality. You can add the same value to each side of an equation.

- For the first equation above, you can add \( x + y \) to the left side and 5 to the right side of the first equation: \( x - y + (x + y) = 1 + 5 \). Now, if you simplify, you get: \( x - y + (x + y) = 1 + 5 \rightarrow 2x = 6 \rightarrow x = 3 \). Now, substitute 3 for the \( x \) in the first equation: \( 3 - y = 1 \). By solving this equation, \( y = 2 \)

Example:

Example 1. What is the value of \( x + y \) in this system of equations?

\[
\begin{align*}
2x + 4y &= 12 \\
4x - 2y &= -16
\end{align*}
\]

Solution: Solving a System of Equations by Elimination:
Multiply the first equation by \((-2)\), then add it to the second equation.

\[
\begin{align*}
-2(2x + 4y &= 12) &\Rightarrow -4x - 8y = -24 \\
4x - 2y &= -16 &\Rightarrow -10y = -40 \Rightarrow y = 4
\end{align*}
\]

Plug in the value of \( y \) into one of the equations and solve for \( x \).

\[
2x + 4(4) = 12 \Rightarrow 2x + 16 = 12 \Rightarrow 2x = -4 \Rightarrow x = -2
\]

Thus, \( x + y = -2 + 4 = 2 \)
Graphing Single–Variable Inequalities

- An inequality compares two expressions using an inequality sign.
- Inequality signs are: “less than" <, "greater than" >, “less than or equal to” ≤, and “greater than or equal to” ≥.
- To graph a single–variable inequality, find the value of the inequality on the number line.
- For less than (<) or greater than (>) draw an open circle on the value of the variable. If there is an equal sign too, then use a filled circle.
- Draw an arrow to the right for greater or to the left for less than.

Examples:

Example 1. Draw a graph for this inequality. \( x > 2 \)

Solution: Since the variable is greater than 2, then we need to find 2 on the number line and draw an open circle on it. Then, draw an arrow to the right.

Example 2. Graph this inequality. \( x \leq -3 \).

Solution: Since the variable is less than or equal to –3, then we need to find –3 in the number line and draw a filled circle on it. Then, draw an arrow to the left.
One-Step Inequalities

- An inequality compares two expressions using an inequality sign.
- Inequality signs are: “less than" <, "greater than" >, “less than or equal to” ≤, and “greater than or equal to” ≥.
- You only need to perform one Math operation to solve the one-step inequalities.
- To solve one-step inequalities, find the inverse (opposite) operation is being performed.
- For dividing or multiplying both sides by negative numbers, flip the direction of the inequality sign.

Examples:

Example 1. Solve this inequality for x. \( x + 5 \geq 4 \)

Solution: The inverse (opposite) operation of addition is subtraction. In this inequality, 5 is added to \( x \). To isolate \( x \) we need to subtract 5 from both sides of the inequality.

Then: \( x + 5 \geq 4 \rightarrow x + 5 - 5 \geq 4 - 5 \rightarrow x \geq -1 \). The solution is: \( x \geq -1 \)

Example 2. Solve the inequality. \( x - 3 > -6 \).

Solution: 3 is subtracted from \( x \). Add 3 to both sides.

\( x - 3 > -6 \rightarrow x - 3 + 3 > -6 + 3 \rightarrow x > -3 \)

Example 3. Solve. \( 4x \leq -8 \).

Solution: 4 is multiplied to \( x \). Divide both sides by 4.

Then: \( 4x \leq -8 \rightarrow \frac{4x}{4} \leq \frac{-8}{4} \rightarrow x \leq -2 \)

Example 4. Solve. \( -3x \leq 6 \).

Solution: -3 is multiplied to \( x \). Divide both sides by -3. Remember when dividing or multiplying both sides of an inequality by negative numbers, flip the direction of the inequality sign.

Then: \( -3x \leq 6 \rightarrow \frac{-3x}{-3} \leq \frac{6}{-3} \rightarrow x \geq -2 \)
Multi–Step Inequalities

- To solve a multi-step inequality, combine “like” terms on one side.
- Bring variables to one side by adding or subtracting.
- Isolate the variable.
- Simplify using the inverse of addition or subtraction.
- Simplify further by using the inverse of multiplication or division.
- For dividing or multiplying both sides by negative numbers, flip the direction of the inequality sign.

Examples:

Example 1. Solve this inequality. \( 8x - 2 \leq 14 \)

Solution: In this inequality, 2 is subtracted from 8x. The inverse of subtraction is addition. Add 2 to both sides of the inequality:
\[
8x - 2 + 2 \leq 14 + 2 \rightarrow 8x \leq 16
\]
Now, divide both sides by 8. Then: \( 8x \leq 16 \rightarrow \frac{8x}{8} \leq \frac{16}{8} \rightarrow x \leq 2 \)
The solution of this inequality is \( x \leq 2 \).

Example 2. Solve this inequality. \( 3x + 9 < 12 \)

Solution: First, subtract 9 from both sides: \( 3x + 9 - 9 < 12 - 9 \)
Then simplify: \( 3x < 3 \)
Now divide both sides by 3. \( \frac{3x}{3} < \frac{3}{3} \rightarrow x < 1 \)

Example 3. Solve this inequality. \(-5x + 3 \geq 8 \)

Solution: First, subtract 3 from both sides:
\[
-5x + 3 - 3 \geq 8 - 3 \rightarrow -5x \geq 5
\]
Divide both sides by \(-5\). Remember that you need to flip the direction of inequality sign. \( -5x \geq 5 \rightarrow \frac{-5x}{-5} \leq \frac{5}{-5} \rightarrow x \leq -1 \)
Chapter 8: Practices

 Solve each equation. (One–Step Equations)

1) \( x + 6 = 3 \rightarrow x = \) 
2) \( 5 = 11 - x \rightarrow x = \) 
3) \( -3 = 8 + x \rightarrow x = \) 
4) \( x - 2 = -7 \rightarrow x = \) 
5) \( -15 = x + 6 \rightarrow x = \)

6) \( 10 - x = -2 \rightarrow x = \) 
7) \( 22 - x = -9 \rightarrow x = \) 
8) \( -4 + x = 28 \rightarrow x = \) 
9) \( 11 - x = -7 \rightarrow x = \) 
10) \( 35 - x = -7 \rightarrow x = \)

 Solve each equation. (Multi–Step Equations)

11) \( 4(x + 2) = 12 \rightarrow x = \) 
12) \( -6(6 - x) = 12 \rightarrow x = \) 
13) \( 5 = -5(x + 2) \rightarrow x = \) 
14) \( -10 = 2(4 + x) \rightarrow x = \) 
15) \( 4(x + 2) = -12, x = \) 
16) \( -6(3 + 2x) = 30, x = \) 
17) \( -3(4 - x) = 12, x = \) 
18) \( -4(6 - x) = 16, x = \)

 Solve each system of equations.

19) \( \begin{cases} x + 6y = 32 \\ x + 3y = 17 \end{cases} \rightarrow \begin{cases} x = \) \\ y = \) 
20) \( \begin{cases} 3x + y = 15 \\ x + 2y = 10 \end{cases} \rightarrow \begin{cases} x = \) \\ y = \) 
21) \( \begin{cases} 3x + 5y = 17 \\ 2x + y = 9 \end{cases} \rightarrow \begin{cases} x = \) \\ y = \) 
22) \( \begin{cases} 5x - 2y = -8 \\ -6x + 2y = 10 \end{cases} \rightarrow \begin{cases} x = \) \\ y = \)
Draw a graph for each inequality.

23) \( x \leq -3 \)

24) \( x > -5 \)

Solve each inequality and graph it.

25) \( x - 2 \geq -2 \)

26) \( 2x - 3 < 9 \)

Solve each inequality.

27) \( x + 13 > 4 \)

28) \( x + 6 > 5 \)

29) \(-12 + 2x \leq 26 \)

30) \(-2 + 8x \leq 14 \)

31) \(6 + 4x \leq 18 \)

32) \(4(x + 3) \geq -12 \)

33) \(2(6 + x) \geq -12 \)

34) \(3(x - 5) < -6 \)

35) \(10 + 5x < -15 \)

36) \(6(6 + x) \geq -18 \)

37) \(2(x - 5) \geq -14 \)

38) \(6(x + 4) < -12 \)

39) \(3(x - 8) \geq -48 \)

40) \(-(6 - 4x) > -30 \)

41) \(2(2 + 2x) > -60 \)

42) \(-3(4 + 2x) > -24 \)
### Chapter 8: Answers

1) $-3$ 
2) $6$ 
3) $-11$ 
4) $-5$ 
5) $-21$ 
6) $12$ 
7) $31$ 
8) $32$ 

9) $18$ 
10) $42$ 
11) $1$ 
12) $8$ 
13) $-3$ 
14) $-9$ 
15) $-5$ 
16) $-4$

17) $8$ 
18) $10$ 
19) $x = 2, y = 5$ 
20) $x = 4, y = 3$ 
21) $x = 4, y = 1$ 
22) $x = -2, y = -1$

23) $x \leq -3$ 
24) $x > -5$ 
25) $x \geq 0$ 
26) $x < 6$ 
27) $x > -9$ 
28) $x > -1$ 
29) $x \leq 19$ 
30) $x \leq 2$ 
31) $x \leq 3$ 
32) $x \geq -6$

33) $x \geq -12$ 
34) $x < 3$ 
35) $x < -5$ 
36) $x \geq -9$ 
37) $x \geq -2$ 
38) $x < -6$

39) $x \geq -8$ 
40) $x > -6$ 
41) $x > -16$ 
42) $x < 2$
Math topics that you’ll learn in this chapter:

- Finding Slope
- Graphing Lines Using Slope– Intercept Form
- Writing Linear Equations
- Finding Midpoint
- Finding Distance of Two Points
- Graphing Linear Inequalities
Finding Slope

- The slope of a line represents the direction of a line on the coordinate plane.

- A coordinate plane contains two perpendicular number lines. The horizontal line is $x$ and the vertical line is $y$. The point at which the two axes intersect is called the origin. An ordered pair $(x, y)$ shows the location of a point.

- A line on a coordinate plane can be drawn by connecting two points.

- To find the slope of a line, we need the equation of the line or two points on the line.

- The slope of a line with two points $A (x_1, y_1)$ and $B (x_2, y_2)$ can be found by using this formula: 

$$ \frac{y_2 - y_1}{x_2 - x_1} = \frac{\text{rise}}{\text{run}} $$

- The equation of a line is typically written as $y = mx + b$ where $m$ is the slope and $b$ is the $y$-intercept.

Examples:

Example 1. Find the slope of the line through these two points:

$A(1, -6)$ and $B(3, 2)$.

Solution: Slope = $\frac{y_2 - y_1}{x_2 - x_1}$. Let $(x_1, y_1)$ be $A(1, -6)$ and $(x_2, y_2)$ be $B(3, 2)$.

(Remember, you can choose any point for $(x_1, y_1)$ and $(x_2, y_2)$).

Then: slope $= \frac{y_2 - y_1}{x_2 - x_1} = \frac{2 - (-6)}{3 - 1} = \frac{8}{2} = 4$

The slope of the line through these two points is 4.

Example 2. Find the slope of the line with equation $y = -2x + 8$

Solution: when the equation of a line is written in the form of $y = mx + b$, the slope is $m$. In this line: $y = -2x + 8$, the slope is $-2$. 

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Graphing Lines Using Slope–Intercept Form

- Slope–intercept form of a line: given the slope \( m \) and the \( y \)-intercept (the intersection of the line and \( y \)-axis) \( b \), then the equation of the line is:

\[
y = mx + b
\]

- To draw the graph of a linear equation in a slope-intercept form on the \( xy \) coordinate plane, find two points on the line by plugging two values for \( x \) and calculating the values of \( y \).

- You can also use the slope \( (m) \) and one point to graph the line.

Example:

Example 1. Sketch the graph of \( y = 2x - 4 \).

Solution: To graph this line, we need to find two points. When \( x \) is zero the value of \( y \) is \(-4\). And when \( x \) is 2 the value of \( y \) is 0.

\[
x = 0 \rightarrow y = 2(0) - 4 = -4, \\
y = 0 \rightarrow 0 = 2x - 4 \rightarrow x = 2
\]

Now, we have two points:
(0, \(-4\)) and (2, 0).
Find the points on the coordinate plane and graph the line. Remember that the slope of the line is 2.
Writing Linear Equations

- The equation of a line in slope-intercept form: \( y = mx + b \)
- To write the equation of a line, first identify the slope.
- Find the y-intercept. This can be done by substituting the slope and the coordinates of a point \((x, y)\) on the line.

Examples:

Example 1. What is the equation of the line that passes through \((3, -4)\) and has a slope of 6?

Solution: The general slope-intercept form of the equation of a line is \( y = mx + b \), where \( m \) is the slope and \( b \) is the y-intercept.
By substitution of the given point and given slope:
\[ y = mx + b \rightarrow -4 = (3)(6) + b \]
So, \( b = -4 - 18 = -22 \), and the required equation is \( y = 6x - 22 \).

Example 2. Write the equation of the line through two points \( A(3,1)\) and \( B(-2,6)\).

Solution: First, find the slope: \( Slop = \frac{y_2 - y_1}{x_2 - x_1} = \frac{6 - 1}{-2 - 3} = \frac{-5}{-5} = 1 \rightarrow m = -1 \)
To find the value of \( b \), use either points and plug in the values of \( x \) and \( y \) in the equation. The answer will be the same: \( y = -x + b \). Let’s check both points.
Then: \( (3,1) \rightarrow y = mx + b \rightarrow 1 = -1(3) + b \rightarrow b = 4 \)
\( (-2,6) \rightarrow y = mx + b \rightarrow 6 = -1(-2) + b \rightarrow b = 4 \).
The y-intercept of the line is 4. The equation of the line is: \( y = -x + 4 \).

Example 3. What is the equation of the line that passes through \((4, -1)\) and has a slope of 4?

Solution: The general slope-intercept form of the equation of a line is \( y = mx + b \), where \( m \) is the slope and \( b \) is the y-intercept. By substitution of the given point and given slope: \( y = mx + b \rightarrow -1 = (4)(4) + b \)
So, \( b = -1 - 16 = -17 \), and the equation of the line is: \( y = 4x - 17 \).
Finding Midpoint

- The middle of a line segment is its midpoint.
- The Midpoint of two endpoints \( A (x_1, y_1) \) and \( B (x_2, y_2) \) can be found using this formula: \( M \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) \)

Examples:

Example 1. Find the midpoint of the line segment with the given endpoints. \((2, -4), (6, 8)\)

Solution: Midpoint \( = \left( \frac{2 + 6}{2}, \frac{-4 + 8}{2} \right) \rightarrow (x_1, y_1) = (2, -4) \) and \( (x_2, y_2) = (6, 8) \)
Midpoint \( = \left( \frac{8}{2}, \frac{4}{2} \right) \rightarrow M(4, 2) \)

Example 2. Find the midpoint of the line segment with the given endpoints. \((-2, 3), (6, -7)\)

Solution: Midpoint \( = \left( \frac{-2 + 6}{2}, \frac{3 - 7}{2} \right) \rightarrow (x_1, y_1) = (-2, 3) \) and \( (x_2, y_2) = (6, -7) \)
Midpoint \( = \left( \frac{4}{2}, \frac{-4}{2} \right) \rightarrow M(2, -2) \)

Example 3. Find the midpoint of the line segment with the given endpoints. \((7, -4), (1, 8)\)

Solution: Midpoint \( = \left( \frac{7 + 1}{2}, \frac{-4 + 8}{2} \right) \rightarrow (x_1, y_1) = (7, -4) \) and \( (x_2, y_2) = (1, 8) \)
Midpoint \( = \left( \frac{8}{2}, \frac{4}{2} \right) \rightarrow M(4, 2) \)

Example 4. Find the midpoint of the line segment with the given endpoints. \((6, 3), (10, -9)\)

Solution: Midpoint \( = \left( \frac{6 + 10}{2}, \frac{3 - 9}{2} \right) \rightarrow (x_1, y_1) = (6, 3) \) and \( (x_2, y_2) = (10, -9) \)
Midpoint \( = \left( \frac{16}{2}, \frac{-6}{2} \right) \rightarrow M(8, -3) \)
Finding Distance of Two Points

- Use the following formula to find the distance of two points with the coordinates A \((x_1, y_1)\) and B \((x_2, y_2)\):

\[ d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \]

Examples:

Example 1. Find the distance between \((4, 2)\) and \((-5, -10)\).

Solution: Use distance of two points formula: 

\[ d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \]

\((x_1, y_1) = (4,2)\) and \((x_2, y_2) = (-5,-10)\). Then:

\[ d = \sqrt{(-5 - 4)^2 + (-10 - 2)^2} \]

\[ d = \sqrt{(-9)^2 + (-12)^2} = \sqrt{81 + 144} = \sqrt{225} = 15 \]

Then: \(d = 15\)

Example 2. Find the distance of two points \((-1,5)\) and \((-3,-6)\).

Solution: Use distance of two points formula: 

\[ d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \]

\((x_1, y_1) = (-1,5),\) and \((x_2, y_2) = (-3,-6)\)

Then:

\[ d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \rightarrow d = \sqrt{(-3 - (-1))^2 + (-6 - (-5))^2} = \sqrt{(-2)^2 + (-11)^2} = \sqrt{4 + 121} = \sqrt{125} = 5\sqrt{5} \]

Then: \(d = 5\sqrt{5}\)

Example 3. Find the distance between \((-6,5)\) and \((-2,2)\).

Solution: Use distance of two points formula: 

\[ d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \]

\((x_1, y_1) = (-6,5)\) and \((x_2, y_2) = (-2,2)\). Then:

\[ d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \]

\[ d = \sqrt{(-2 - (-6))^2 + (2 - 5)^2} = \sqrt{(4)^2 + (-3)^2} = \sqrt{16 + 9} = \sqrt{25} = 5 \]
Graphing Linear Inequalities

- To graph a linear inequality, first draw a graph of the "equals" line.
- Use a dash line for less than (<) and greater than (>) signs and a solid line for less than and equal to (≤) and greater than and equal to (≥).
- Choose a testing point. (it can be any point on both sides of the line.)
- Put the value of \((x, y)\) of that point in the inequality. If that works, that part of the line is the solution. If the values don’t work, then the other part of the line is the solution.

Example:

Sketch the graph of inequality: \(y < 2x + 4\)

**Solution:** To draw the graph of \(y < 2x + 4\), you first need to graph the line:
\[y = 2x + 4\]
Since there is a less than (<) sign, draw a dash line.
The slope is 2 and \(y\)-intercept is 4.
Then, choose a testing point and substitute the value of \(x\) and \(y\) from that point into the inequality. The easiest point to test is the origin: \((0, 0)\)
\[(0,0) \rightarrow y < 2x + 4 \rightarrow 0 < 2(0) + 4 \rightarrow 0 < 4\]
This is correct! 0 is less than 4. So, this part of the line (on the right side) is the solution of this inequality.
Chapter 9: Practices

✍ Find the slope of each line.

1) \( y = x - 5 \)  
2) \( y = 2x + 6 \)  
3) \( y = -5x - 8 \)  
4) Line through \((2, 6)\) and \((5, 0)\)  
5) Line through \((8, 0)\) and \((-4, 3)\)  
6) Line through \((-2, -4)\) and \((-4, 8)\)

✍ Sketch the graph of each line. (Using Slope–Intercept Form)

7) \( y = x + 4 \)  
8) \( y = 2x - 5 \)

✍ Solve.

9) What is the equation of a line with slope 4 and intercept 16? ______________

10) What is the equation of a line with slope 3 and passes through point \((1, 5)\)? ______________

11) What is the equation of a line with slope \(-5\) and passes through point \((-2, 7)\)? ______________

12) The slope of a line is \(-4\) and it passes through point \((-6, 2)\). What is the equation of the line? ______________

13) The slope of a line is \(-3\) and it passes through point \((-3, -6)\). What is the equation of the line? ______________
Sketch the graph of each linear inequality.

14) \( y > 2x - 2 \)

15) \( y < -x + 3 \)

Find the midpoint of the line segment with the given endpoints.

16) \((5, 0), (1, 4)\)

17) \((2, 3), (4, 7)\)

18) \((8, 1), (2, 5)\)

19) \((5, 10), (3, 6)\)

20) \((4, -1), (-2, 7)\)

21) \((2, -5), (4, 1)\)

22) \((7, 6), (-5, 2)\)

23) \((-2, 8), (4, -6)\)

Find the distance between each pair of points.

24) \((-2, 8), (-6, 8)\)

25) \((4, -4), (14, 20)\)

26) \((-1, 9), (-5, 6)\)

27) \((0, 3), (4, 3)\)

28) \((0, -2), (5, 10)\)

29) \((4, 3), (7, -1)\)

30) \((2, 6), (10, -9)\)

31) \((3, 3), (6, -1)\)

32) \((-2, -12), (14, 18)\)

33) \((2, -2), (12, 22)\)
Chapter 9: Answers

1) 1  
2) 2  
3) −5  
4) −2  
5) −\frac{1}{4}

7) \( y = x + 4 \)

8) \( y = 2x - 5 \)

9) \( y = 4x + 16 \)  
10) \( y = 3x + 2 \)

11) \( y = -5x - 3 \)  
12) \( y = -4x - 22 \)

13) \( y = -3x - 15 \)

14) \( y > 2x - 2 \)

15) \( y < -x + 3 \)

16) (3, 2)  
17) (3, 5)  
18) (5, 3)  
19) (4, 8)  
20) (1, 3)  
21) (3, −2)  
22) (1, 4)  
23) (1, 1)  
24) 4  
25) 26  
26) 5  
27) 4  
28) 13  
29) 5  
30) 17  
31) 5  
32) 34  
33) 26
Math topics that you’ll learn in this chapter:

- Simplifying Polynomials
- Adding and Subtracting Polynomials
- Multiplying Monomials
- Multiplying and Dividing Monomials
- Multiplying a Polynomial and a Monomial
- Multiplying Binomials
- Factoring Trinomials
Simplifying Polynomials

- To simplify Polynomials, find “like” terms. (they have same variables with same power).
- Use “FOIL”. (First–Out–In–Last) for binomials:

\[(x + a)(x + b) = x^2 + (b + a)x + ab\]
- Add or Subtract “like” terms using order of operation.

Examples:

Example 1. Simplify this expression. \[x(4x + 7) - 2x = \]

Solution: Use Distributive Property: \[x(4x + 7) = 4x^2 + 7x \]
Now, combine like terms: \[x(4x + 7) - 2x = 4x^2 + 7x - 2x = 4x^2 + 5x \]

Example 2. Simplify this expression. \[(x + 3)(x + 5) = \]

Solution: First, apply the FOIL method: \[(a + b)(c + d) = ac + ad + bc + bd \]
\[(x + 3)(x + 5) = x^2 + 5x + 3x + 15 \]
Now combine like terms: \[x^2 + 5x + 3x + 15 = x^2 + 8x + 15 \]

Example 3. Simplify this expression. \[2x(x - 5) - 3x^2 + 6x = \]

Solution: Use Distributive Property: \[2x(x - 5) = 2x^2 - 10x \]
Then: \[2x(x - 5) - 3x^2 + 6x = 2x^2 - 10x - 3x^2 + 6x \]
Now combine like terms: \[2x^2 - 3x^2 = -x^2, \text{ and } -10x + 6x = -4x \]
The simplified form of the expression: \[2x^2 - 10x - 3x^2 + 6 = -x^2 - 4x \]
Adding and Subtracting Polynomials

- Adding polynomials is just a matter of combining like terms, with some order of operations considerations thrown in.
- Be careful with the minus signs, and don't confuse addition and multiplication!
- For subtracting polynomials, sometimes you need to use the Distributive Property: \( a(b + c) = ab + ac, a(b - c) = ab - ac \)

Examples:

Example 1. Simplify the expressions. \((x^2 - 2x^3) - (x^3 - 3x^2) = \)

Solution: First, use Distributive Property:
\[-(x^3 - 3x^2) = -x^3 + 3x^2\]
\[\rightarrow (x^2 - 2x^3) - (x^3 - 3x^2) = x^2 - 2x^3 - x^3 + 3x^2\]
Now combine like terms: \(-2x^3 - x^3 = -3x^3\) and \(x^2 + 3x^2 = 4x^2\)
Then: \((x^2 - 2x^3) - (x^3 - 3x^2) = x^2 - 2x^3 - x^3 + 3x^2 = -3x^3 + 4x^2\)

Example 2. Add expressions. \((3x^3 - 5) + (4x^3 - 2x^2) = \)

Solution: Remove parentheses:
\[(3x^3 - 5) + (4x^3 - 2x^2) = 3x^3 - 5 + 4x^3 - 2x^2\]
Now combine like terms: \(3x^3 - 5 + 4x^3 - 2x^2 = 7x^3 - 2x^2 - 5\)

Example 3. Simplify the expressions. \((-4x^2 - 2x^3) - (5x^2 + 2x^3) = \)

Solution: First, use Distributive Property:
\[-(5x^2 + 2x^3) = -5x^2 - 2x^3 \rightarrow (-4x^2 - 2x^3) - (5x^2 + 2x^3)\]
\[= -4x^2 - 2x^3 - 5x^2 - 2x^3\]
Now combine like terms and write in standard form:
\[-4x^2 - 2x^3 - 5x^2 - 2x^3 = -4x^3 - 9x^2\]
Multiplying Monomials

- A monomial is a polynomial with just one term: Examples: \(2x\) or \(7y^2\).
- When you multiply monomials, first multiply the coefficients (a number placed before and multiplying the variable) and then multiply the variables using multiplication property of exponents.

\[ x^a \times x^b = x^{a+b} \]

Examples:

Example 1. Multiply expressions. \(2xy^3 \times 6x^4y^2\)

Solution: Find the same variables and use multiplication property of exponents:

\[ x^a \times x^b = x^{a+b} \]
\[ x \times x^4 = x^{1+4} = x^5 \text{ and } y^3 \times y^2 = y^{3+2} = y^5 \]

Then, multiply coefficients and variables: \(2xy^3 \times 6x^4y^2 = 12x^5y^5\)

Example 2. Multiply expressions. \(7a^3b^8 \times 3a^6b^4\) =

Solution: Use the multiplication property of exponents: \(x^a \times x^b = x^{a+b}\)

\[ a^3 \times a^6 = a^{3+6} = a^9 \text{ and } b^8 \times b^4 = b^{8+4} = b^{12} \]

Then: \(7a^3b^8 \times 3a^6b^4 = 21a^9b^{12} \)

Example 3. Multiply. \(5x^2y^4z^3 \times 4x^4y^7z^5\)

Solution: Use the multiplication property of exponents: \(x^a \times x^b = x^{a+b}\)

\[ x^2 \times x^4 = x^{2+4} = x^6, \ y^4 \times y^7 = y^{4+7} = y^{11} \text{ and } z^3 \times z^5 = z^{3+5} = z^8 \]

Then: \(5x^2y^4z^3 \times 4x^4y^7z^5 = 20x^6y^{11}z^8\)

Example 4. Simplify. \((-6a^7b^4)(4a^8b^5)\) =

Solution: Use the multiplication property of exponents: \(x^a \times x^b = x^{a+b}\)

\[ a^7 \times a^8 = a^{7+8} = a^{15} \text{ and } b^4 \times b^5 = b^{4+5} = b^9 \]

Then: \((-6a^7b^4)(4a^8b^5) = -24a^{15}b^9\)
Chapter 10: Polynomials

**Multiplying and Dividing Monomials**

- When you divide or multiply two monomials, you need to divide or multiply their coefficients and then divide or multiply their variables.

- In case of exponents with the same base, for Division, subtract their powers, for Multiplication, add their powers.

- Exponent’s Multiplication and Division rules:

\[
x^a \times x^b = x^{a+b}, \quad \frac{x^a}{x^b} = x^{a-b}
\]

**Examples:**

**Example 1.** Multiply expressions. \((3x^5)(9x^4) =
\)

**Solution:** Use multiplication property of exponents:

\[
x^a \times x^b = x^{a+b} \rightarrow x^5 \times x^4 = x^9
\]

Then: \((3x^5)(9x^4) = 27x^9\)

**Example 2.** Divide expressions. \(\frac{12x^4y^6}{6xy^2} =
\)

**Solution:** Use division property of exponents:

\[
x^a = x^{a-b} \rightarrow \frac{x^4}{x} = x^{4-1} = x^3 \quad \text{and} \quad \frac{y^6}{y^2} = y^{6-2} = y^4
\]

Then: \(\frac{12x^4y^6}{6xy^2} = 2x^3y^4\)

**Example 3.** Divide expressions. \(\frac{49a^6b^9}{7a^3b^4} =
\)

**Solution:** Use division property of exponents:

\[
x^a = x^{a-b} \rightarrow \frac{a^6}{a^3} = a^{6-3} = a^3 \quad \text{and} \quad \frac{b^9}{b^4} = b^{9-4} = b^5
\]

Then: \(\frac{49a^6b^9}{7a^3b^4} = 7a^3b^5\)

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Multiplying a Polynomial and a Monomial

- When multiplying monomials, use the product rule for exponents.
  \[ x^a \times x^b = x^{a+b} \]

- When multiplying a monomial by a polynomial, use the distributive property.
  \[ a \times (b + c) = a \times b + a \times c = ab + ac \]
  \[ a \times (b - c) = a \times b - a \times c = ab - ac \]

Examples:

Example 1. Multiply expressions. \( 6x(2x + 5) \)

Solution: Use Distributive Property:
\[ 6x(2x + 5) = 6x \times 2x + 6x \times 5 = 12x^2 + 30x \]

Example 2. Multiply expressions. \( x(3x^2 + 4y^2) \)

Solution: Use Distributive Property:
\[ x(3x^2 + 4y^2) = x \times 3x^2 + x \times 4y^2 = 3x^3 + 4xy^2 \]

Example 3. Multiply. \( -x(-2x^2 + 4x + 5) \)

Solution: Use Distributive Property:
\[ -x(-2x^2 + 4x + 5) = (-x)(-2x^2) + (-x) \times (4x) + (-x) \times (5) = \]
Now simplify:
\[ (-x)(-2x^2) + (-x) \times (4x) + (-x) \times (5) = 2x^3 - 4x^2 - 5x \]
Multiplying Binomials

- A binomial is a polynomial that is the sum or the difference of two terms, each of which is a monomial.
- To multiply two binomials, use the “FOIL” method. (First–Out–In–Last)

\[(x + a)(x + b) = x \times x + x \times b + a \times x + a \times b = x^2 + bx + ax + ab\]

Examples:

Example 1. Multiply Binomials. \((x + 3)(x - 2) =\)

**Solution:** Use “FOIL”. (First–Out–In–Last):

\((x + 3)(x - 2) = x^2 - 2x + 3x - 6\)

Then combine like terms: \(x^2 - 2x + 3x - 6 = x^2 + x - 6\)

Example 2. Multiply. \((x + 6)(x + 4) =\)

**Solution:** Use “FOIL”. (First–Out–In–Last):

\((x + 6)(x + 4) = x^2 + 4x + 6x + 24\)

Then simplify: \(x^2 + 4x + 6x + 24 = x^2 + 10x + 24\)

Example 3. Multiply. \((x + 5)(x - 7) =\)

**Solution:** Use “FOIL”. (First–Out–In–Last):

\((x + 5)(x - 7) = x^2 - 7x + 5x - 35\)

Then simplify: \(x^2 - 7x + 5x - 35 = x^2 - 2x - 35\)

Example 4. Multiply Binomials. \((x - 9)(x - 5) =\)

**Solution:** Use “FOIL”. (First–Out–In–Last):

\((x - 9)(x - 5) = x^2 - 5x - 9x + 45\)

Then combine like terms: \(x^2 - 5x - 9x + 45 = x^2 - 14x + 45\)
Factoring Trinomials

To factor trinomials, you can use following methods:

- “FOIL”: \((x + a)(x + b) = x^2 + (b + a)x + ab\)
- “Difference of Squares”:
  \[a^2 - b^2 = (a + b)(a - b)\]
  \[a^2 + 2ab + b^2 = (a + b)(a + b)\]
  \[a^2 - 2ab + b^2 = (a - b)(a - b)\]
- “Reverse FOIL”: \(x^2 + (b + a)x + ab = (x + a)(x + b)\)

Examples:

Example 1. Factor this trinomial. \(x^2 - 2x - 8\)

Solution: Break the expression into groups. You need to find two numbers that their product is \(-8\) and their sum is \(-2\). (remember “Reverse FOIL”: \(x^2 + (b + a)x + ab = (x + a)(x + b)\)). Those two numbers are 2 and \(-4\). Then: \(x^2 - 2x - 8 = (x^2 + 2x) + (-4x - 8)\)

Now factor out \(x\) from \(x^2 + 2x\) : \(x(x + 2)\), and factor out \(-4\) from \(-4x - 8: -4(x + 2)\); Then: \((x^2 + 2x) + (-4x - 8) = x(x + 2) - 4(x + 2)\)

Now factor out like term: \((x + 2)\). Then: \((x + 2)(x - 4)\)

Example 2. Factor this trinomial. \(x^2 - 2x - 24\)

Solution: Break the expression into groups: \((x^2 + 4x) + (-6x - 24)\)

Now factor out \(x\) from \(x^2 + 4x\) : \(x(x + 4)\), and factor out \(-6\) from \(-6x - 24: -6(x + 4)\); Then: \((x + 4) - 6(x + 4)\), now factor out like term: \((x + 4) \rightarrow x(x + 4) - 6(x + 4) = (x + 4)(x - 6)\)
Chapter 10: Practices

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1) \(3(6x + 4) = \)

2) \(5(3x - 8) = \)

3) \(x(7x + 2) + 9x = \)

4) \(6x(x + 3) + 5x = \)

5) \(6x(3x + 1) - 5x = \)

6) \(x(3x - 4) + 3x^2 - 6 = \)

7) \(x^2 - 5 - 3x(x + 8) = \)

8) \(2x^2 + 7 - 6x(2x + 5) = \)

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9) \((x^2 + 3) + (2x^2 - 4) = \)

10)\((3x^2 - 6x) - (x^2 + 8x) = \)

11)\((4x^3 - 3x^2) + (2x^3 - 5x^2) = \)

12)\((6x^3 - 7x) - (5x^3 - 3x) = \)

13)\((10x^3 + 4x^2) + (14x^2 - 8) = \)

14)\((4x^3 - 9) - (3x^3 - 7x^2) = \)

15)\((9x^3 + 3x) - (6x^3 - 4x) = \)

16)\((7x^3 - 5x) - (3x^3 + 5x) = \)

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17) \(3x^2 \times 8x^3 = \)

18) \(2x^4 \times 9x^3 = \)

19) \(-4a^4b \times 2ab^3 = \)

20) \((-7x^3yz) \times (3xy^2z^4) = \)

21) \(-2a^5bc \times 6ab^4 = \)

22) \(9u^3t^2 \times (-2ut) = \)

23) \(12x^2z \times 3xy^3 = \)

24) \(11x^3z \times 5xy^5 = \)

25) \(-6a^3bc \times 5a^4b^3 = \)

26) \(-4x^6y^2 \times (-12xy) = \)
Simplify each expression. (Multiplying and Dividing Monomials)

27) \((7x^2y^3)(3x^4y^2) =

28) \((6x^3y^2)(4x^4y^3) =

29) \((10x^8y^5)(3x^5y^7) =

30) \((15a^3b^2)(2a^3b^8) =

31) \frac{42x^4y^2}{6x^3y} =

32) \frac{49x^5y^6}{7x^2y} =

33) \frac{63x^{15}y^{10}}{9x^8y^6} =

34) \frac{35x^8y^{12}}{5x^4y^{8}} =

Find each product. (Multiplying a Polynomial and a Monomial)

35) \(3x(5x - y) =

36) \(2x(4x + y) =

37) \(7x(x - 3y) =

38) \(x(2x^2 + 2x - 4) =

39) \(5x(3x^2 + 8x + 2) =

40) \(7x(2x^2 - 9x - 5) =

Find each product. (Multiplying Binomials)

41) \((x - 3)(x + 3) =

42) \((x - 6)(x + 6) =

43) \((x + 10)(x + 4) =

44) \((x - 6)(x + 7) =

45) \((x + 2)(x - 5) =

46) \((x - 10)(x + 3) =

Factor each trinomial.

47) \(x^2 + 6x + 8 =

48) \(x^2 + 3x - 10 =

49) \(x^2 + 2x - 48 =

50) \(x^2 - 10x + 80 =

51) \(2x^2 - 7x + 12 =

52) \(3x^2 - 10x + 3 =

Chapter 10: Answers

1) \(18x + 12\)  
2) \(15x - 40\)  
3) \(7x^2 + 11x\)  
4) \(6x^2 + 23x\)  
5) \(18x^2 + x\)  
6) \(6x^2 - 4x - 6\)  
7) \(-2x^2 - 24x - 5\)  
8) \(-10x^2 - 30x + 7\)  
9) \(3x^2 - 1\)  
10) \(2x^2 - 14x\)  
11) \(6x^3 - 8x^2\)  
12) \(x^3 - 4x\)  
13) \(10x^3 + 18x^2 - 8\)  
14) \(x^3 + 7x^2 - 9\)  
15) \(3x^3 + 7x\)  
16) \(4x^3 - 10x\)  
17) \(24x^5\)  
18) \(18x^7\)  

19) \(-8a^5b^4\)  
20) \(-21x^4y^3z^5\)  
21) \(-12a^7b^5c\)  
22) \(-18u^4t^3\)  
23) \(36x^3y^3z\)  
24) \(55x^4y^5z\)  
25) \(-30a^7b^4c\)  
26) \(48x^7y^3\)  
27) \(21x^6y^5\)  
28) \(24x^7y^5\)  
29) \(30x^{13}y^{12}\)  
30) \(30a^6b^{10}\)  
31) \(7xy\)  
32) \(7x^3y^5\)  
33) \(7x^7y^4\)  
34) \(7x^4y^4\)  
35) \(15x^2 - 3xy\)  
36) \(8x^2 + 2xy\)  
37) \(7x^2 - 21xy\)  
38) \(2x^3 + 2x^2 - 4x\)  
39) \(15x^3 + 40x^2 + 10x\)  
40) \(14x^3 - 63x^2 - 35x\)  
41) \(x^2 - 9\)  
42) \(x^2 - 36\)  
43) \(x^2 + 14x + 40\)  
44) \(x^2 + x - 42\)  
45) \(x^2 - 3x - 10\)  
46) \(x^2 - 7x - 30\)  
47) \((x + 4)(x + 2)\)  
48) \((x + 5)(x - 2)\)  
49) \((x - 6)(x + 8)\)  
50) \((x - 8)(x - 2)\)  
51) \((2x - 4)(x - 3)\)  
52) \((3x - 1)(x - 3)\)
Geometry and Solid Figures

Math topics that you’ll learn in this chapter:

- The Pythagorean Theorem
- Complementary and Supplementary angles
- Parallel lines and Transversals
- Triangles
- Special Right Triangles
- Polygons
- Circles
- Trapezoids
- Cubes
- Rectangle Prisms
- Cylinder
The Pythagorean Theorem

- You can use the Pythagorean Theorem to find a missing side in a right triangle.
- In any right triangle: \( a^2 + b^2 = c^2 \)

Examples:

Example 1. Right triangle ABC (not shown) has two legs of lengths 3 cm (AB) and 4 cm (AC). What is the length of the hypotenuse of the triangle (side BC)?

Solution: Use Pythagorean Theorem: \( a^2 + b^2 = c^2 \), \( a = 3 \), and \( b = 4 \)
Then: \( a^2 + b^2 = c^2 \rightarrow 3^2 + 4^2 = c^2 \rightarrow 9 + 16 = c^2 \rightarrow 25 = c^2 \rightarrow c = \sqrt{25} = 5 \)
The length of the hypotenuse is 5 cm.

Example 2. Find the hypotenuse of this triangle.

Solution: Use Pythagorean Theorem: \( a^2 + b^2 = c^2 \)
Then: \( a^2 + b^2 = c^2 \rightarrow 8^2 + 6^2 = c^2 \rightarrow 64 + 36 = c^2 \)
\( c^2 = 100 \rightarrow c = \sqrt{100} = 10 \)

Example 3. Find the length of the missing side in this triangle.

Solution: Use Pythagorean Theorem: \( a^2 + b^2 = c^2 \)
Then: \( a^2 + b^2 = c^2 \rightarrow 12^2 + b^2 = 15^2 \rightarrow 144 + b^2 = 225 \rightarrow \\
b^2 = 225 - 144 \rightarrow b^2 = 81 \rightarrow b = \sqrt{81} = 9 \)
Complementary and Supplementary angles

- Two angles with a sum of 90 degrees are called complementary angles.

- Two angles with a sum of 180 degrees are Supplementary angles.

Examples:

Example 1. Find the missing angle.

Solution: Notice that the two angles form a right angle. This means that the angles are complementary, and their sum is 90.

Then: \(18 + x = 90 \rightarrow x = 90^\circ - 18^\circ = 72^\circ\)

The missing angle is 72 degrees. \(x = 72^\circ\)

Example 2. Angles Q and S are supplementary. What is the measure of angle Q if angle S is 35 degrees?

Solution: Q and S are supplementary \(\rightarrow Q + S = 180 \rightarrow Q + 35 = 180 \rightarrow Q = 180 - 35 = 145\)
Parallel lines and Transversals

- When a line (transversal) intersects two parallel lines in the same plane, eight angles are formed. In the following diagram, a transversal intersects two parallel lines. Angles 1, 3, 5, and 7 are congruent. Angles 2, 4, 6, and 8 are also congruent.

- In the following diagram, the following angles are supplementary angles (their sum is 180):
  - Angles 1 and 8
  - Angles 2 and 7
  - Angles 3 and 6
  - Angles 4 and 5

Example:

Example 1. In the following diagram, two parallel lines are cut by a transversal. What is the value of $x$?

Solution: The two angles $3x - 15$ and $2x + 7$ are equivalent.

That is: $3x - 15 = 2x + 7$

Now, solve for $x$:

$3x - 15 + 15 = 2x + 7 + 15$

$3x - 15 = 2x + 22$

$3x - 2x = 2x + 22 - 2x$

$x = 22$
Chapter 11: Geometry and Solid Figures

**Triangles**

- In any triangle, the sum of all angles is 180 degrees.
- Area of a triangle = \( \frac{1}{2} (\text{base} \times \text{height}) \)

![Diagram of a triangle with labeled sides and height]

**Examples:**

What is the area of the following triangles?

**Example 1.**

**Solution:** Use the area formula:

Area = \( \frac{1}{2} (\text{base} \times \text{height}) \)

base = 14 and height = 10

Area = \( \frac{1}{2} (14 \times 10) = \frac{1}{2} (140) = 70 \)

**Example 2.**

**Solution:** Use the area formula:

Area = \( \frac{1}{2} (\text{base} \times \text{height}) \)

base = 16 and height = 8; Area = \( \frac{1}{2} (16 \times 8) = \frac{128}{2} = 64 \)

**Example 3.** What is the missing angle in this triangle?

**Solution:**

In any triangle, the sum of all angles is 180 degrees.

Let \( x \) be the missing angle.

Then: 55 + 80 + \( x \) = 180

\( \rightarrow 135 + x = 180 \rightarrow x = 180 - 135 = 45 \)

The missing angle is 45 degrees.
Special Right Triangles

- A special right triangle is a triangle whose sides are in a particular ratio. Two special right triangles are $45^\circ - 45^\circ - 90^\circ$ and $30^\circ - 60^\circ - 90^\circ$ triangles.

- In a special $45^\circ - 45^\circ - 90^\circ$ triangle, the three angles are $45^\circ$, $45^\circ$ and $90^\circ$. The lengths of the sides of this triangle are in the ratio of $1:1:\sqrt{2}$.

- In a special triangle $30^\circ - 60^\circ - 90^\circ$, the three angles are $30^\circ - 60^\circ - 90^\circ$. The lengths of this triangle are in the ratio of $1:\sqrt{3}:2$.

Examples:

**Example 1.** Find the length of the hypotenuse of a right triangle if the length of the other two sides are both 4 inches.

**Solution:** this is a right triangle with two equal sides. Therefore, it must be a $45^\circ - 45^\circ - 90^\circ$ triangle. Two equivalent sides are 4 inches. The ratio of sides: $x:x:x\sqrt{2}$

The length of the hypotenuse is $4\sqrt{2}$ inches. $x:x:x\sqrt{2} \rightarrow 4:4:4\sqrt{2}$

**Example 2.** The length of the hypotenuse of a triangle is 6 inches. What are the lengths of the other two sides if one angle of the triangle is $30^\circ$?

**Solution:** The hypotenuse is 6 inches and the triangle is a $30^\circ - 60^\circ - 90^\circ$ triangle.

Then, one side of the triangle is 3 (it’s half the side of the hypotenuse) and the other side is $3\sqrt{3}$. (it’s the smallest side times $\sqrt{3}$)

$x:x\sqrt{3}:2x \rightarrow x = 3 \rightarrow x:x\sqrt{3}:2x = 3:3\sqrt{3}:6$
Chapter 11: Geometry and Solid Figures

Polygons

- The perimeter of a square = $4 \times side = 4s$

- The perimeter of a rectangle = $2(width + length)$

- The perimeter of a trapezoid = $a + b + c + d$

- The perimeter of a regular hexagon = $6a$

- The perimeter of a parallelogram = $2(l + w)$

Examples:

Example 1. Find the perimeter of following regular hexagon.

Solution: Since the hexagon is regular, all sides are equal.
Then: The perimeter of the hexagon = $6 \times (one \ side)$
The perimeter of the hexagon = $6 \times (one \ side) = 6 \times 8 = 48 \ m$

Example 2. Find the perimeter of following trapezoid.

Solution: The perimeter of a trapezoid = $a + b + c + d$
The perimeter of the trapezoid = $7 + 8 + 8 + 10 = 33 \ ft$
Circles

- In a circle, variable \( r \) is usually used for the radius and \( d \) for diameter.
- \( \text{Area of a circle} = \pi r^2 \) (\( \pi \) is about 3.14)
- \( \text{Circumference of a circle} = 2\pi r \)

Examples:

Example 1. Find the area of this circle.

Solution:
Use area formula: \( \text{Area} = \pi r^2 \)
\( r = 6 \text{ in} \rightarrow \text{Area} = \pi (6)^2 = 36\pi, \pi = 3.14 \)
Then: \( \text{Area} = 36 \times 3.14 = 113.04 \text{ in}^2 \)

Example 2. Find the Circumference of this circle.

Solution:
Use Circumference formula: \( \text{Circumference} = 2\pi r \)
\( r = 8 \text{ cm} \rightarrow \text{Circumference} = 2\pi (8) = 16\pi \)
\( \pi = 3.14 \) Then: \( \text{Circumference} = 16 \times 3.14 = 50.24 \text{ cm} \)

Example 3. Find the area of the circle.

Solution:
Use area formula: \( \text{Area} = \pi r^2 \),
\( r = 9 \text{ in} \) then: \( \text{Area} = \pi (9)^2 = 81\pi, \pi = 3.14 \)
Then: \( \text{Area} = 81 \times 3.14 = 254.34 \text{ in}^2 \)
Trapezoids

- A quadrilateral with at least one pair of parallel sides is a trapezoid.

- Area of a trapezoid = \( \frac{1}{2} h(b_1 + b_2) \)

Examples:

Example 1. Calculate the area of this trapezoid.

Solution:
Use area formula: \( A = \frac{1}{2} h(b_1 + b_2) \)

\( b_1 = 6 \text{ cm} \), \( b_2 = 10 \text{ cm} \) and \( h = 12 \text{ cm} \)

Then: \( A = \frac{1}{2} (12)(10 + 6) = 6(16) = 96 \text{ cm}^2 \)

Example 2. Calculate the area of this trapezoid.

Solution:
Use area formula: \( A = \frac{1}{2} h(b_1 + b_2) \)

\( b_1 = 10 \text{ cm} \), \( b_2 = 18 \text{ cm} \) and \( h = 14 \text{ cm} \)

Then: \( A = \frac{1}{2} (14)(10 + 18) = 7(28) = 196 \text{ cm}^2 \)
Cubes

- A cube is a three-dimensional solid object bounded by six square sides.
- Volume is the measure of the amount of space inside of a solid figure, like a cube, ball, cylinder or pyramid.
- The volume of a cube \( = (\text{one side})^3 \)
- The surface area of a cube \( = 6 \times (\text{one side})^2 \)

Examples:

Example 1. Find the volume and surface area of this cube.

**Solution:** Use volume formula: \( \text{volume} = (\text{one side})^3 \)
Then: \( \text{volume} = (\text{one side})^3 = (3)^3 = 27 \text{ cm}^3 \)
Use surface area formula:
\( \text{surface area of cube: } 6(\text{one side})^2 = 6(3)^2 = 6(9) = 54 \text{ cm}^2 \)

Example 2. Find the volume and surface area of this cube.

**Solution:** Use volume formula: \( \text{volume} = (\text{one side})^3 \)
Then: \( \text{volume} = (\text{one side})^3 = (6)^3 = 216 \text{ cm}^3 \)
Use surface area formula:
\( \text{surface area of cube: } 6(\text{one side})^2 = 6(6)^2 = 6(36) = 216 \text{ cm}^2 \)

Example 3. Find the volume and surface area of this cube.

**Solution:** Use volume formula: \( \text{volume} = (\text{one side})^3 \)
Then: \( \text{volume} = (\text{one side})^3 = (8)^3 = 512 \text{ m}^3 \)
Use surface area formula:
\( \text{surface area of cube: } 6(\text{one side})^2 = 6(8)^2 = 6(64) = 384 \text{ m}^2 \)
Rectangular Prisms

- A rectangular prism is a solid 3-dimensional object with six rectangular faces.

- The volume of a Rectangular prism = \( \text{Length} \times \text{Width} \times \text{Height} \)

\[
\text{Volume} = l \times w \times h
\]

\[
\text{Surface area} = 2 \times (wh + lw + lh)
\]

Examples:

Example 1. Find the volume and surface area of this rectangular prism.

Solution: Use volume formula: \( \text{Volume} = l \times w \times h \)

Then: \( \text{Volume} = 7 \times 5 \times 9 = 315 \text{ m}^3 \)

Use surface area formula: \( \text{Surface area} = 2 \times (wh + lw + lh) \)

Then: \( \text{Surface area} = 2 \times ((5 \times 9) + (7 \times 5) + (7 \times 9)) \)

\[
= 2 \times (45 + 35 + 63) = 2 \times 143 = 286 \text{ m}^2
\]

Example 2. Find the volume and surface area of this rectangular prism.

Solution: Use volume formula: \( \text{Volume} = l \times w \times h \)

Then: \( \text{Volume} = 9 \times 6 \times 12 = 648 \text{ m}^3 \)

Use surface area formula: \( \text{Surface area} = 2 \times (wh + lw + lh) \)

Then: \( \text{Surface area} = 2 \times ((6 \times 12) + (9 \times 6) + (9 \times 12)) \)

\[
= 2 \times (72 + 54 + 108) = 2 \times (234) = 468 \text{ m}^2
\]
**Cylinder**

- A cylinder is a solid geometric figure with straight parallel sides and a circular or oval cross-section.
- \( \text{Volume of a Cylinder} = \pi (\text{radius})^2 \times \text{height}, \pi \approx 3.14 \)
- \( \text{Surface area of a cylinder} = 2\pi r^2 + 2\pi rh \)

**Examples:**

**Example 1.** Find the volume and Surface area of the follow Cylinder.

**Solution:** Use volume formula:
\[
\text{Volume} = \pi (\text{radius})^2 \times \text{height}
\]
Then: \( \text{Volume} = \pi (4)^2 \times 10 = 16\pi \times 10 = 160\pi \)
\( \pi = 3.14 \) then: \( \text{Volume} = 160\pi = 160 \times 3.14 = 502.4 \text{ cm}^3 \)
Use surface area formula: \( \text{Surface area} = 2\pi r^2 + 2\pi rh \)
Then: \(2\pi(4)^2 + 2\pi(4)(10) = 2\pi(16) + 2\pi(40) = 32\pi + 80\pi = 112\pi \)
\( \pi = 3.14 \) Then: \( \text{Surface area} = 112 \times 3.14 = 351.68 \text{ cm}^2 \)

**Example 2.** Find the volume and Surface area of the follow Cylinder.

**Solution:** Use volume formula:
\[
\text{Volume} = \pi (\text{radius})^2 \times \text{height}
\]
Then: \( \text{Volume} = \pi (5)^2 \times 8 = \pi 25 \times 8 = 200\pi \)
\( \pi = 3.14 \) then: \( \text{Volume} = 200\pi = 628 \text{ cm}^3 \)
Use surface area formula: \( \text{Surface area} = 2\pi r^2 + 2\pi rh \)
Then: \(2\pi(5)^2 + 2\pi(5)(8) = 2\pi(25) + 2\pi(40) = 50\pi + 80\pi = 130\pi \)
\( \pi = 3.14 \) then: \( \text{Surface area} = 130 \times 3.14 = 408.2 \text{ cm}^2 \)
Chapter 11: Practices

1) Find the missing side?

2) Find the measure of the unknown angle in each triangle.

3) Find the area of each triangle.

4) Find the perimeter or circumference of each shape.

(Images of geometric shapes and measurements are shown, but not transcribed here.)
Find the area of each trapezoid.

17) \[ \text{Area} = \frac{(10 \text{ m} + 7 \text{ m}) \times 14 \text{ m}}{2} \]
18) \[ \text{Area} = \frac{(10 \text{ cm} + 8 \text{ cm}) \times 15 \text{ cm}}{2} \]
19) \[ \text{Area} = \frac{(8 \text{ ft} + 6 \text{ ft}) \times 13 \text{ ft}}{2} \]
20) \[ \text{Area} = \frac{(8 \text{ cm} + 6 \text{ cm}) \times 12 \text{ cm}}{2} \]

Find the volume of each cube.

21) \[ \text{Volume} = 3 \text{ cm}^3 \]
22) \[ \text{Volume} = 10 \text{ ft}^3 \]
23) \[ \text{Volume} = 5 \text{ in}^3 \]
24) \[ \text{Volume} = 9 \text{ miles}^3 \]

Find the volume of each Rectangular Prism.

25) \[ \text{Volume} = 8 \text{ cm} \times 6 \text{ cm} \times 4 \text{ cm} = 192 \text{ cm}^3 \]
26) \[ \text{Volume} = 10 \text{ m} \times 8 \text{ m} \times 3 \text{ m} = 240 \text{ m}^3 \]
27) \[ \text{Volume} = 12 \text{ in} \times 7 \text{ in} \times 4 \text{ in} = 336 \text{ in}^3 \]

Find the volume of each Cylinder. Round your answer to the nearest tenth. (\( \pi = 3.14 \))

28) \[ \text{Volume} = \pi \times 8 \text{ cm} \times 14 \text{ cm} \approx 804.2 \text{ cm}^3 \]
29) \[ \text{Volume} = \pi \times 6 \text{ m} \times 8 \text{ m} \approx 113.1 \text{ m}^3 \]
30) \[ \text{Volume} = \pi \times 9 \text{ cm} \times 14 \text{ cm} \approx 376.9 \text{ cm}^3 \]
**Chapter 11: Answers**

1) 4  
11) $64 \text{ cm}^2$  
21) $27 \text{ cm}^3$

2) 15  
12) $90 \text{ in}^2$  
22) $1,000 \text{ ft}^3$

3) 6  
13) $44 \text{ cm}$  
23) $125 \text{ in}^3$

4) 13  
14) $30 \text{ ft}$  
24) $729 \text{ mi}^3$

5) 50  
15) $10 \pi \approx 31.4 \text{ in}$  
25) $192 \text{ cm}^3$

6) 76  
16) $24 \text{ m}$  
26) $240 \text{ m}^3$

7) 84  
17) $84 \text{ m}^2$  
27) $336 \text{ in}^3$

8) 70  
18) $100 \text{ cm}^2$  
28) $2,813.44 \text{ cm}^3$

9) 30  
19) $63 \text{ ft}^2$  
29) $904.32 \text{ m}^3$

10) 49.5  
20) $60 \text{ cm}^2$  
30) $3,560.76 \text{ cm}^3$
CHAPTER 12

Statistics

Math topics that you’ll learn in this chapter:

- Mean, Median, Mode, and Range of the Given Data
- Pie Graph
- Probability Problems
- Permutations and Combinations
Mean, Median, Mode, and Range of the Given Data

- Mean: \( \frac{\text{sum of the data}}{\text{total number of data entries}} \)

- Mode: the value in the list that appears most often

- Median: is the middle number of a group of numbers arranged in order by size.

- Range: the difference of the largest value and smallest value in the list

Examples:

Example 1. What is the mode of these numbers? 5, 6, 8, 6, 8, 5, 3, 5

Solution: Mode: the value in the list that appears most often. Therefore, the mode is number 5. There are three number 5 in the data.

Example 2. What is the median of these numbers? 6, 11, 15, 10, 17, 20, 7

Solution: Write the numbers in order: 6, 7, 10, 11, 15, 17, 20. The median is the number in the middle. Therefore, the median is 11.

Example 3. What is the mean of these numbers? 7, 2, 3, 2, 4, 8, 7, 5

Solution: Mean: \( \frac{\text{sum of the data}}{\text{total number of data entries}} = \frac{7+2+3+2+4+8+7+5}{8} = \frac{38}{8} = 4.75 \)

Example 4. What is the range in this list? 3, 7, 12, 6, 15, 20, 8

Solution: Range is the difference of the largest value and smallest value in the list. The largest value is 20 and the smallest value is 3. Then: \( 20 - 3 = 17 \)
Pie Graph

- A Pie Chart is a circle chart divided into sectors, each sector represents the relative size of each value.
- Pie charts represent a snapshot of how a group is broken down into smaller pieces.

Example:

A library has 750 books that include Mathematics, Physics, Chemistry, English and History. Use the following graph to answer the questions.

Example 1. What is the number of Mathematics books?

Solution: Number of total books = 750
Percent of Mathematics books = 28% = 0.28
Then, the number of Mathematics books: $0.28 \times 750 = 210$

Example 2. What is the number of History books?

Solution: Number of total books = 750
Percent of History books = 12% = 0.12
Then: $0.12 \times 750 = 90$

Example 3. What is the number of Chemistry books?

Solution: Number of total books = 750
Percent of Chemistry books = 22% = 0.22
Then: $0.22 \times 750 = 165$
Probability Problems

- Probability is the likelihood of something happening in the future. It is expressed as a number between zero (can never happen) to 1 (will always happen).

- Probability can be expressed as a fraction, a decimal, or a percent.

- Probability formula: \( \text{Probability} = \frac{\text{number of desired outcomes}}{\text{number of total outcomes}} \)

Examples:

Example 1. Anita’s trick-or-treat bag contains 10 pieces of chocolate, 16 suckers, 16 pieces of gum, 22 pieces of licorice. If she randomly pulls a piece of candy from her bag, what is the probability of her pulling out a piece of sucker?

Solution: Probability = \( \frac{\text{number of desired outcomes}}{\text{number of total outcomes}} \)

\[
\text{Probability of pulling out a piece of sucker} = \frac{16}{10 + 16 + 16 + 22} = \frac{16}{64} = \frac{1}{4}
\]

Example 2. A bag contains 20 balls: four green, five black, eight blue, a brown, a red and one white. If 19 balls are removed from the bag at random, what is the probability that a brown ball has been removed?

Solution: If 19 balls are removed from the bag at random, there will be one ball in the bag. The probability of choosing a brown ball is 1 out of 20. Therefore, the probability of not choosing a brown ball is 19 out of 20 and the probability of having not a brown ball after removing 19 balls is the same. The answer is: \( \frac{19}{20} \)
Permutations and Combinations

Factorials are products, indicated by an exclamation mark. For example, 
4! = 4 × 3 × 2 × 1 (Remember that 0! is defined to be equal to 1)

- **Permutations:** The number of ways to choose a sample of \( k \) elements from a set of \( n \) distinct objects where order does matter, and replacements are not allowed. For a permutation problem, use this formula:
  \[
nP_k = \frac{n!}{(n-k)!}\]

- **Combination:** The number of ways to choose a sample of \( r \) elements from a set of \( n \) distinct objects where order does not matter, and replacements are not allowed. For a combination problem, use this formula:
  \[
nC_r = \frac{n!}{r!(n-r)!}\]

Examples:

**Example 1.** How many ways can the first and second place be awarded to 7 people?

*Solution:* Since the order matters, (the first and second place are different!) we need to use permutation formula where \( n \) is 7 and \( k \) is 2. Then:
  \[
  \frac{n!}{(n-k)!} = \frac{7!}{(7-2)!} = \frac{7!}{5!} = \frac{7 \times 6 \times 5!}{5!} \text{, remove } 5! \text{ from both sides of the fraction. Then: } \frac{7 \times 6 \times 5!}{5!} = 7 \times 6 = 42
  \]

**Example 2.** How many ways can we pick a team of 3 people from a group of 8?

*Solution:* Since the order doesn’t matter, we need to use a combination formula where \( n \) is 8 and \( r \) is 3.
Then:
  \[
  \frac{n!}{r!(n-r)!} = \frac{8!}{3!(8-3)!} = \frac{8!}{3! \times 5!} = \frac{8 \times 7 \times 6 \times 5!}{3! \times 5!} = \frac{8 \times 7 \times 6}{3! \times 2 \times 1} = \frac{8 \times 7 \times 6}{6} = \frac{336}{6} = 56
  \]
Chapter 12: Practices

Find the values of the Given Data.

1) 6, 11, 5, 3, 6
   Mode: _____  Range: _____
   Mean: _____  Median: _____

2) 4, 9, 1, 9, 6, 7
   Mode: _____  Range: _____
   Mean: _____  Median: _____

3) 10, 3, 6, 10, 4, 15
   Mode: _____  Range: _____
   Mean: _____  Median: _____

4) 12, 4, 8, 9, 3, 12, 15
   Mode: _____  Range: _____
   Mean: _____  Median: _____

The circle graph below shows all Bob’s expenses for last month. Bob spent $790 on his Rent last month.

5) How much did Bob’s total expenses last month? ________

6) How much did Bob spend for foods last month? ________

7) How much did Bob spend for his bills last month? ________

8) How much did Bob spend on his car last month? ________
Solve.

9) Bag A contains 8 red marbles and 6 green marbles. Bag B contains 5 black marbles and 7 orange marbles. What is the probability of selecting a green marble at random from bag A? What is the probability of selecting a black marble at random from Bag B?

_______________   ______________

Solve.

10) Susan is baking cookies. She uses sugar, flour, butter, and eggs. How many different orders of ingredients can she try? ______________

11) Jason is planning for his vacation. He wants to go to museum, go to the beach, and play volleyball. How many different ways of ordering are there for him? ______________

12) In how many ways can a team of 6 basketball players choose a captain and co-captain? ______________

13) How many ways can you give 5 balls to your 8 friends? ______________

14) A professor is going to arrange her 5 students in a straight line. In how many ways can she do this? ______________

15) In how many ways can a teacher chooses 12 out of 15 students? ______________
Chapter 12: Answers

1) Mode: 6, Range: 8, Mean: 6.2, Median: 6
2) Mode: 9, Range: 8, Mean: 6, Median: 6.5
3) Mode: 10, Range: 12, Mean: 8, Median: 8
4) Mode: 12, Range: 12, Mean: 9, Median: 9
5) $1,975
6) $158
7) $730.75
8) $197.50
9) $ \frac{3}{7}, \frac{5}{12}

10) 24
11) 6

12) 30 (it’s a permutation problem)
13) 56 (it’s a combination problem)
14) 120

15) 455 (it’s a combination problem)
Math topics that you’ll learn in this chapter:

- Function Notation and Evaluation
- Adding and Subtracting Functions
- Multiplying and Dividing Functions
- Composition of Functions
- Function Inverses
Function Notation and Evaluation

- Functions are mathematical operations that assign unique outputs to given inputs.
- Function notation is the way a function is written. It is meant to be a precise way of giving information about the function without a rather lengthy written explanation.
- The most popular function notation is \( f(x) \) which is read "\( f \) of \( x \)". Any letter can name a function. for example: \( g(x), h(x), \) etc.
- To evaluate a function, plug in the input (the given value or expression) for the function’s variable (place holder, \( x \)).

Examples:

**Example 1.** Evaluate: \( f(x) = x + 6 \), find \( f(2) \)

*Solution:* Substitute \( x \) with 2:
Then: \( f(x) = x + 6 \rightarrow f(2) = 2 + 6 \rightarrow f(2) = 8 \)

**Example 2.** Evaluate: \( w(x) = 3x - 1 \), find \( w(4) \).

*Solution:* Substitute \( x \) with 4:
Then: \( w(x) = 3x - 1 \rightarrow w(4) = 3(4) - 1 = 12 - 1 = 11 \)

**Example 3.** Evaluate: \( f(x) = 2x^2 + 4 \), find \( f(-1) \).

*Solution:* Substitute \( x \) with \(-1\):
Then: \( f(x) = 2x^2 + 4 \rightarrow f(-1) = 2(-1)^2 + 4 \rightarrow f(-1) = 2 + 4 = 6 \)

**Example 4.** Evaluate: \( h(x) = 4x^2 - 9 \), find \( h(2a) \).

Solution: Substitute \( x \) with \( 3a \):
Then: \( h(x) = 4x^2 - 9 \rightarrow h(2a) = 4(2a)^2 - 9 \rightarrow h(2a) = 4(4a^2) - 9 = 16a^2 - 9 \)
Adding and Subtracting Functions

- Just like we can add and subtract numbers and expressions, we can add or subtract two functions and simplify or evaluate them. The result is a new function.

- For two functions \( f(x) \) and \( g(x) \), we can create two new functions:
  \[
  (f + g)(x) = f(x) + g(x) \quad \text{and} \quad (f - g)(x) = f(x) - g(x)
  \]

Examples:

Example 1. \( g(x) = 2x - 2, f(x) = x + 1 \), Find: \( (g + f)(x) \)

Solution: \( (g + f)(x) = g(x) + f(x) \)

Then: \( (g + f)(x) = (2x - 2) + (x + 1) = 2x - 2 + x + 1 = 3x - 1 \)

Example 2. \( f(x) = 4x - 3, g(x) = 2x - 4 \), Find: \( (f - g)(x) \)

Solution: \( (f - g)(x) = f(x) - g(x) \)

Then: \( (f - g)(x) = (4x - 3) - (2x - 4) = 4x - 3 - 2x + 4 = 2x + 1 \)

Example 3. \( g(x) = x^2 + 2, f(x) = x + 5 \), Find: \( (g + f)(x) \)

Solution: \( (g + f)(x) = g(x) + f(x) \)

Then: \( (g + f)(x) = (x^2 + 2) + (x + 5) = x^2 + x + 7 \)

Example 4. \( f(x) = 5x^2 - 3, g(x) = 3x + 6 \), Find: \( (f - g)(3) \)

Solution: \( (f - g)(x) = f(x) - g(x) \)

Then: \( (f - g)(x) = (5x^2 - 3) - (3x + 6) = 5x^2 - 3 - 3x - 6 = 5x^2 - 3x - 9 \)

Substitute \( x \) with 3: \( (f - g)(3) = 5(3)^2 - 3(3) - 9 = 45 - 9 - 9 = 27 \)
Multiplying and Dividing Functions

- Just like we can multiply and divide numbers and expressions, we can multiply and divide two functions and simplify or evaluate them.

- For two functions \( f(x) \) and \( g(x) \), we can create two new functions:

\[
(f \cdot g)(x) = f(x) \cdot g(x) \quad \text{and} \quad \left( \frac{f}{g} \right)(x) = \frac{f(x)}{g(x)}
\]

**Examples:**

**Example 1.** \( g(x) = x + 3, f(x) = x + 4 \), Find: \( (g \cdot f)(x) \)

**Solution:**

\[
(g \cdot f)(x) = g(x) \cdot f(x) = (x + 3)(x + 4) = x^2 + 4x + 3x + 12 = x^2 + 7x + 12
\]

**Example 2.** \( f(x) = x + 6, h(x) = x - 9 \), Find: \( \left( \frac{f}{h} \right)(x) \)

**Solution:**

\[
\left( \frac{f}{h} \right)(x) = \frac{f(x)}{h(x)} = \frac{x+6}{x-9}
\]

**Example 3.** \( g(x) = x + 7, f(x) = x - 3 \), Find: \( (g \cdot f)(2) \)

**Solution:**

\[
(g \cdot f)(x) = g(x) \cdot f(x) = (x + 7)(x - 3) = x^2 - 3x + 7x - 21 \quad g(x) \cdot f(x) = x^2 + 4x - 21
\]

Substitute \( x \) with \( 2 \):

\[
(g \cdot f)(2) = (2)^2 + 4(2) - 21 = 4 + 8 - 21 = -9
\]

**Example 4.** \( f(x) = x + 3, h(x) = 2x - 4 \), Find: \( \left( \frac{f}{h} \right)(3) \)

**Solution:**

\[
\left( \frac{f}{h} \right)(x) = \frac{f(x)}{h(x)} = \frac{x+3}{2x-4}
\]

Substitute \( x \) with \( 3 \):

\[
\left( \frac{f}{h} \right)(3) = \frac{3+3}{2(3)-4} = \frac{6}{2} = 3
\]
Composition of Functions

- “Composition of functions" simply means combining two or more functions in a way where the output from one function becomes the input for the next function.
- The notation used for composition is: \((fog)(x) = f(g(x))\) and is read "\(f \) composed with \(g \) of \(x\)" or "\(f \) of \(g \) of \(x\)."

Examples:

Example 1. Using \(f(x) = 2x + 3\) and \(g(x) = 5x\), find: \((fog)(x)\)

Solution: \((fog)(x) = f(g(x))\). Then: \((fog)(x) = f(g(x)) = f(5x)\)

Now find \(f(5x)\) by substituting \(x\) with \(5x\) in \(f(x)\) function.

Then: \(f(x) = 2x + 3; (x \rightarrow 5x) \rightarrow f(5x) = 2(5x) + 3 = 10x + 3\)

Example 2. Using \(f(x) = 3x - 1\) and \(g(x) = 2x - 2\), find: \((gof)(5)\)

Solution: \((gof)(x) = f(g(x))\). Then: \((gof)(x) = g(f(x)) = g(3x - 1)\),

Now substitute \(x\) in \(g(x)\) by \(3x - 1\).

Then: \(g(3x - 1) = 2(3x - 1) - 2 = 6x - 2 - 2 = 6x - 4\)

Substitute \(x\) with 5: \((gof)(5) = g(f(x)) = 6x - 4 = 6(5) - 4 = 26\)

Example 3. Using \(f(x) = 2x^2 - 5\) and \(g(x) = x + 3\), find: \(f(g(3))\)

Solution: First, find \(g(3): g(x) = x + 3 \rightarrow g(3) = 3 + 3 = 6\)

Then: \(f(g(3)) = f(6)\). Now, find \(f(6)\) by substituting \(x\) with 6 in \(f(x)\) function.

\(f(g(3)) = f(6) = 2(6)^2 - 5 = 2(36) - 5 = 67\)
Function Inverses

- An inverse function is a function that reverses another function: if the function $f$ applied to an input $x$ gives a result of $y$, then applying its inverse function $g$ to $y$ gives the result $x$. $f(x) = y$ if and only if $g(y) = x$

- The inverse function of $f(x)$ is usually shown by $f^{-1}(x)$.

Examples:

Example 1. Find the inverse of the function: $f(x) = 2x - 1$

Solution: First, replace $f(x)$ with $y$: $y = 2x - 1$, Then, replace all $x$’s with $y$ and all $y$’s with $x$: $x = 2y - 1$, Now, solve for $y$: $x = 2y - 1 \rightarrow x + 1 = 2y \rightarrow \frac{1}{2}x + \frac{1}{2} = y$

Finally replace $y$ with $f^{-1}(x)$: $f^{-1}(x) = \frac{1}{2}x + \frac{1}{2}$

Example 2. Find the inverse of the function: $g(x) = \frac{1}{5}x + 3$

Solution: $g(x) = \frac{1}{5}x + 3 \rightarrow y = \frac{1}{5}x + 3 \rightarrow$ replace all $x$’s with $y$ and all $y$’s with $x$ $x = \frac{1}{5}y + 3$, solve for $y$: $\rightarrow x - 3 = \frac{1}{5}y \rightarrow 5(x - 3) = y \rightarrow y = 5x - 15 \rightarrow g^{-1}(x) = 5x - 15$

Example 3. Find the inverse of the function: $h(x) = \sqrt{x} + 6$

Solution: $h(x) = \sqrt{x} + 6 \rightarrow y = \sqrt{x} + 6$, replace all $x$’s with $y$ and all $y$’s with $x$ $\rightarrow x = \sqrt{y} + 6 \rightarrow x - 6 = \sqrt{y} \rightarrow (x - 6)^2 = (\sqrt{y})^2 \rightarrow x^2 - 12x + 36 = y$ $\rightarrow h^{-1}(x) = x^2 - 12x + 36$
Chapter 13: Functions Operations

Evaluate each function.

1) \( g(n) = 2n + 5 \), find \( g(2) \)
2) \( h(x) = 5n - 9 \), find \( h(4) \)
3) \( k(n) = 10 - 6n \), find \( k(2) \)

4) \( g(x) = -5x + 6 \), find \( g(-2) \)
5) \( k(n) = -8n + 3 \), find \( k(-6) \)
6) \( w(n) = -2n - 9 \), find \( w(-5) \)

Perform the indicated operation.

7) \( f(x) = x + 6 \)
   \( g(x) = 3x + 2 \)
   Find \( (f - g)(x) \)

8) \( g(x) = x - 9 \)
   \( f(x) = 2x - 1 \)
   Find \( (g - f)(x) \)

9) \( h(t) = 5t + 6 \)
   \( g(t) = 2t + 4 \)
   Find \( (h + g)(x) \)

10) \( g(a) = -6a + 1 \)
    \( f(a) = 3a^2 - 3 \)
    Find \( (g + f)(5) \)

11) \( g(x) = 7x - 1 \)
    \( h(x) = -4x^2 + 2 \)
    Find \( (g - h)(-3) \)

12) \( h(x) = -x^2 - 1 \)
    \( g(x) = -7x - 1 \)
    Find \( (h - g)(-5) \)
Perform the indicated operation.

13) \( g(x) = x + 3 \)
\[ f(x) = x + 1 \]
Find \((g \cdot f)(x)\)

14) \( f(x) = 4x \)
\[ h(x) = x - 6 \]
Find \((f \cdot h)(x)\)

15) \( g(a) = a - 8 \)
\[ h(a) = 4a - 2 \]
Find \((g \cdot h)(3)\)

Using \( f(x) = 4x + 3 \) and \( g(x) = x - 7 \), find:

19) \( g(f(2)) = \)_____
20) \( g(f(-2)) = \)_____
21) \( f(g(4)) = \)_____
22) \( f(f(7)) = \)_____
23) \( g(f(5)) = \)_____
24) \( g(f(-5)) = \)_____

Find the inverse of each function.

25) \( f(x) = \frac{1}{x} - 6 \rightarrow f^{-1}(x) = \)
26) \( g(x) = \frac{7}{x-3} \rightarrow g^{-1}(x) = \)
27) \( h(x) = \frac{x+9}{3} \rightarrow h^{-1}(x) = \)
28) \( h(x) = \frac{2x-10}{4} \rightarrow h^{-1}(x) = \)
29) \( f(x) = \frac{-15+x}{3} \rightarrow f^{-1}(x) = \)
30) \( s(x) = \sqrt{x} - 2 \rightarrow s^{-1}(x) = \)
Chapter 13: Answers

1) 9
2) 11
3) −2
4) 16
5) 51
6) 1
7) −2x – 4
8) −x – 8
9) 7t + 10
10) 43
11) 12
12) −60
13) \(x^2 + 4x + 3\)
14) \(4x^2 - 24x\)
15) −50
16) \(\frac{10}{11}\)
17) \(-\frac{29}{3}\)
18) \(\frac{5}{3}\)
19) 4
20) −12
21) −9
22) 127
23) 16
24) −24
25) \(f^{-1}(x) = \frac{1}{x+6}\)
26) \(g^{-1}(x) = \frac{-7+3x}{x}\)
27) \(h^{-1}(x) = 3x - 9\)
28) \(h^{-1}(x) = 2x + 5\)
29) \(f^{-1}(x) = 3x + 15\)
30) \(s^{-1}(x) = x^2 + 4x + 4\)
Chapter 14: Quadratic

Math topics that you’ll learn in this chapter:

- Solving a Quadratic Equation
- Graphing Quadratic Functions
- Solving Quadratic Inequalities
- Graphing Quadratic Inequalities
Solving a Quadratic Equation

- Write the equation in the form of: \( ax^2 + bx + c = 0 \)
- Factorize the quadratic, set each factor equal to zero and solve.
- Use quadratic formula if you couldn’t factorize the quadratic.
- Quadratic formula: \( x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} \)

Examples:

Find the solutions of each quadratic function.

Example 1. \( x^2 + 7x + 12 = 0 \)

Solution: Factor the quadratic by grouping. We need to find two numbers whose sum is 7 (from 7x) and whose product is 12. Those numbers are 3 and 4. Then: \( x^2 + 7x + 12 = 0 \rightarrow x^2 + 3x + 4x + 12 = 0 \rightarrow (x^2 + 3x) + (4x + 12) = 0 \). Now, find common factors: \( x(x + 3) \) and \( 4(x + 3) \). We have two expressions \( x(x + 3) \) and \( 4(x + 3) \) and their common factor is \( x + 3 \). Then: \( (x^2 + 3x) + (4x + 12) = 0 \rightarrow x(x + 3) + 4(x + 3) = 0 \rightarrow (x + 3)(x + 4) = 0 \).

The product of two expressions is 0. Then: \( (x + 3) = 0 \rightarrow x = -3 \) or \( (x + 4) = 0 \rightarrow x = -4 \)

Example 2. \( x^2 + 5x + 6 = 0 \)

Solution: Use quadratic formula: \( x_{1,2} = \frac{-b \pm \sqrt{b^2-4ac}}{2a} \), \( a = 1, b = 5 \) and \( c = 6 \)

Then: \( x = \frac{-5 \pm \sqrt{5^2-4 \cdot 1 \cdot 6}}{2(1)} \), \( x_1 = \frac{-5 + \sqrt{5^2-4 \cdot 1 \cdot 6}}{2(1)} = -2 \), \( x_2 = \frac{-5 - \sqrt{5^2-4 \cdot 1 \cdot 6}}{2(1)} = -3 \)
Chapter 14: Quadratic

Graphing Quadratic Functions

- Quadratic functions in vertex form: \( y = a(x - h)^2 + k \) where \((h,k)\) is the vertex of the function. The axis of symmetry is \( x = h \)

- Quadratic functions in standard form: \( y = ax^2 + bx + c \) where \( x = -\frac{b}{2a} \) is the value of \( x \) in the vertex of the function.

- To graph a quadratic function, first find the vertex, then substitute some values for \( x \) and solve for \( y \). (Remember that the graph of a quadratic function is a U-shaped curve and it is called “parabola”.)

Example:

Sketch the graph of \( y = (x + 2)^2 - 3 \)

**Solution:** Quadratic functions in vertex form: \( y = a(x - h)^2 + k \) and \((h,k)\) is the vertex. Then, the vertex of \( y = (x + 2)^2 - 3 \) is \((-2, -3)\).

Substitute zero for \( x \) and solve for \( y \):
\[
y = (0 + 2)^2 - 3 = 1.
\]
The \( y \) Intercept is \((0,1)\).

Now, you can simply graph the quadratic function. Notice that quadratic function is a U-shaped curve.
Solving Quadratic Inequalities

- A quadratic inequality is one that can be written in the standard form of \( ax^2 + bx + c > 0 \) (or substitute \(<, \leq, \text{ or } \geq \) for \( > \)).

- Solving a quadratic inequality is like solving equations. We need to find the solutions (the zeroes).

- To solve quadratic inequalities, first find quadratic equations. Then choose a test value between zeroes. Finally, find interval(s), such as \( > 0 \) or \( < 0 \).

Examples:

Example 1. Solve quadratic inequality. \( x^2 + x - 6 > 0 \)

Solution: First solve \( x^2 + x - 6 = 0 \) by factoring. Then: \( x^2 + x - 6 = 0 \rightarrow (x - 2)(x + 3) = 0 \). The product of two expressions is 0. Then: \( (x - 2) = 0 \rightarrow x = 2 \) or \( (x + 3) = 0 \rightarrow x = -3 \). Now, choose a value between 2 and \(-3\). Let’s choose 0. Then: \( x = 0 \rightarrow x^2 + x - 6 > 0 \rightarrow (0)^2 + (0) - 6 > 0 \rightarrow -6 > 0 \)

\(-6\) is not greater than 0. Therefore, all values between 2 and \(-3\) are NOT the solution of this quadratic inequality. The solution is: \( x > 2 \) and \( x < -3 \). To represent the solution, we can use interval notation, in which solution sets are indicated with parentheses or brackets. The solutions \( x > 2 \) and \( x < -3 \) represented as: \((\infty, -3) \cup (2, \infty)\)

Example 2. Solve quadratic inequality. \( x^2 - 2x - 8 \geq 0 \)

Solution: First solve: \( x^2 - 2x - 8 = 0 \), Factor: \( x^2 - 2x - 8 = 0 \rightarrow (x - 4)(x + 2) = 0 \).

\(-2\) and 4 are the solutions. Choose a point between \(-2\) and 4. Let’s choose 0. Then: \( x = 0 \rightarrow x^2 - 2x - 8 \geq 0 \rightarrow (0)^2 - 2(0) - 8 \geq 0 \rightarrow -8 \geq 0 \).

This is NOT true. So, the solution is: \( x \leq -2 \) or \( x \geq 4 \) (using interval notation the solution is: \((\infty, -2] \cup [4, \infty)\)
Graphing Quadratic Inequalities

- A quadratic inequality is in the form
  \[ y > ax^2 + bx + c \] (or substitute <, ≤, or ≥ for >).
- To graph a quadratic inequality, start by graphing the quadratic parabola. Then fill in the region either inside or outside of it, depending on the inequality.
- Choose a testing point and check the solution section.

Example:

Sketch the graph of \( y = 2x^2 \)

Solution: First, graph the quadratic \( y = 2x^2 \)
Since the inequality sign is >, we need to use dash lines.

Now, choose a testing point inside the parabola. Let’s choose (0,2).

\[ y > 2x^2 \rightarrow 2 > 2(0)^2 \rightarrow 2 > 0 \]

This is true. So, inside the parabola is the solution section.
Chapter 14: Practices

هى Solve each equation by factoring or using the quadratic formula.

1) \( x^2 - 4x - 32 = 0 \)  
   \[ \underline{\quad} \]

2) \( x^2 - 2x - 63 = 0 \)  
   \[ \underline{\quad} \]

3) \( x^2 + 17x + 72 = 0 \)  
   \[ \underline{\quad} \]

4) \( x^2 + 14x + 48 = 0 \)  
   \[ \underline{\quad} \]

5) \( x^2 + 5x - 24 = 0 \)  
   \[ \underline{\quad} \]

6) \( x^2 + 15x + 36 = 0 \)  
   \[ \underline{\quad} \]

هى Sketch the graph of each function.

7) \( y = (x + 1)^2 - 2 \)

8) \( y = (x - 1)^2 + 3 \)
Solve each quadratic inequality.

9) \( x^2 - 4 < 0 \)

10) \( x^2 - 9 > 0 \)

11) \( x^2 - 5x - 6 < 0 \)

12) \( x^2 + 8x - 20 > 0 \)

13) \( x^2 + 10x - 24 \geq 0 \)

14) \( x^2 + 17x + 72 \leq 0 \)

Sketch the graph of each quadratic inequality.

15) \( y < -2x^2 \)

16) \( y > 3x^2 \)
Chapter 14: Answers

1) \(x^2 - 4x - 32 = 0\)
   \[x = 8, x = -4\]

2) \(x^2 - 2x - 63 = 0\)
   \[x = 9, x = -7\]

3) \(x^2 + 17x + 72 = 0\)
   \[x = -9, x = -8\]

4) \(x^2 + 14x + 48 = 0\)
   \[x = -6, x = -8\]

5) \(x^2 + 5x - 24 = 0\)
   \[x = 3, x = -8\]

6) \(x^2 + 15x + 36 = 0\)
   \[x = -12, x = -3\]

7) \(y = (x + 1)^2 - 2\)

8) \(y = (x - 1)^2 + 3\)

9) \(x^2 - 4 < 0\)
   \[-2 < x < 2\]

10) \(x^2 - 9 > 0\)
    \[-3 < x < 3\]

11) \(x^2 - 5x - 6 < 0\)
    \[-1 < x < 6\]

12) \(x^2 + 8x - 20 > 0\)
    \[x < -10 \text{ or } x > 2\]

13) \(x^2 + 10x - 24 \geq 0\)
    \[x \leq -12 \text{ or } x \geq 2\]

14) \(x^2 + 17x + 72 \leq 0\)
    \[-9 \leq x \leq -8\]

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15) $y < -2x^2$

16) $y > 3x^2$
Math topics that you’ll learn in this chapter:

- Adding and Subtracting Complex Numbers
- Multiplying and Dividing Complex Numbers
- Rationalizing Imaginary Denominators
Adding and Subtracting Complex Numbers

- A complex number is expressed in the form $a + bi$, where $a$ and $b$ are real numbers, and $i$, which is called an imaginary number, is a solution of the equation $x^2 = -1$.
- For adding complex numbers:
  \[(a + bi) + (c + di) = (a + c) + (b + d)i\]
- For subtracting complex numbers:
  \[(a + bi) - (c + di) = (a - c) + (b - d)i\]

Examples:

Example 1. Solve: $(8 + 4i) + (6 - 2i)$

Solution: Remove parentheses: $(8 + 4i) + (6 - 2i) = 8 + 4i + 6 - 2i$
Combine like terms: $8 + 4i + 6 - 2i = 14 + 2i$

Example 2. Solve: $(10 + 8i) + (8 - 3i)$

Solution: Remove parentheses: $(10 + 8i) + (8 - 3i) = 10 + 8i + 8 - 3i$
Group like terms: $10 + 8i + 8 - 3i = 18 + 5i$

Example 3. Solve: $(-5 - 3i) - (2 + 4i)$

Solution: Remove parentheses by multiplying $-1$ to the second parentheses:
\[(-5 - 3i) - (2 + 4i) = -5 - 3i - 2 - 4i\]
Combine like terms: $-5 - 3i - 2 - 4i = -7 - 7i$
Multiplying and Dividing Complex Numbers

- You can use FOIL (First-Out-In-Last) method or the following rule to multiply imaginary numbers. Remember that: $i^2 = -1$

$$ (a + bi) + (c + di) = (ac - bd) + (ad + bc)i $$

- To divide complex numbers, you need to find the conjugate of the denominator. Conjugate of $(a + bi)$ is $(a - bi)$.

- Dividing complex numbers:

$$ \frac{a + bi}{c + di} = \frac{a + bi}{c + di} \times \frac{c - di}{c - di} = \frac{ac + bd}{c^2 + d^2} + \frac{bc - ad}{c^2 + d^2}i $$

Examples:

Example 1. Solve: $\frac{6-2i}{2+i}$

**Solution:** The conjugate of $(2 + i)$ is $(2 - i)$. Use the rule for dividing complex numbers:

$$ \frac{a + bi}{c + di} = \frac{a + bi}{c + di} \times \frac{c - di}{c - di} = \frac{ac + bd}{c^2 + d^2} + \frac{bc - ad}{c^2 + d^2}i \rightarrow $$

$$ \frac{6-2i}{2+i} \times \frac{2-i}{2-i} = \frac{6 \times (2) + (-2)(1)}{2^2 + (1)^2} + \frac{-2 \times 2 - (6)(1)}{2^2 + (1)^2}i = \frac{10}{5} + \frac{-10}{5}i = 2 - 2i $$

Example 2. Solve: $(2 - 3i)(6 - 3i)$

**Solution:** Use the multiplication of imaginary numbers rule:

$$ (a + bi) + (c + di) = (ac - bd) + (ad + bc)i $$

$$(2 \times 6 - (-3)(-3)) + (2(-3) + (-3 \times 6)i = 3 - 24i$$

Example 3. Solve: $\frac{3-2i}{4+i}$

**Solution:** Use the rule for dividing complex numbers:

$$ \frac{a + bi}{c + di} = \frac{a + bi}{c + di} \times \frac{c - di}{c - di} = $$

$$ \frac{ac + bd}{c^2 + d^2} + \frac{bc - ad}{c^2 + d^2}i \rightarrow \frac{3-2i}{4+i} \times \frac{4-i}{4-i} = \frac{(3 \times 4 + (-2)(1) + (-2 \times 4 - 3 \times 1)i}{4^2 + 1^2} = \frac{10 - 11i}{17} = \frac{10}{17} - \frac{11}{17}i$$
Rationalizing Imaginary Denominators

- Step 1: Find the conjugate (it’s the denominator with different sign between the two terms).
- Step 2: Multiply numerator and denominator by the conjugate.
- Step 3: Simplify if needed.

Examples:

Example 1. Solve: $\frac{4-3i}{6i}$

Solution: Multiply both numerator and denominator by $\frac{i}{i}$:

$$\frac{4-3i}{6i} = \frac{(4-3i)(i)}{6i(i^2)} = \frac{(4)(i) - (3i)(i)}{6(-1)} = \frac{4i - 3(-1)}{-6} = \frac{4i + 3}{-6} = -\frac{1}{2} - \frac{3}{2}i$$

Example 2. Solve: $\frac{6i}{2-i}$

Solution: Multiply both numerator and denominator by the conjugate

$$\frac{2+i}{2+i} \cdot \frac{6i(2+i)}{(2-i)(2+i)} = \text{Apply complex arithmetic rule: } (a + bi)(a - bi) = a^2 + b^2$$

$$2^2 + (-1)^2 = 5, \text{ then: } \frac{6i(2+i)}{(2-i)(2+i)} = \frac{-6+12i}{5} = -\frac{6}{5} + \frac{12}{5}i$$

Example 3. Solve: $\frac{8-2i}{2i}$

Solution: Factor 2 from both sides: $\frac{8-2i}{2i} = \frac{2(4-i)}{2i}$, divide both sides by 2:

$$\frac{2(4-i)}{2i} = \frac{(4-i)}{i}$$

Multiply both numerator and denominator by $\frac{i}{i}$:

$$\frac{(4-i)}{i} \times \frac{i}{i} = \frac{(4i-i^2)}{i^2} = \frac{1+4i}{-1} = -1 - 4i$$
Chapter 15: Complex Numbers

Chapter 15: Practices

Evaluate.

1) \((-5i) - (7i) = \)
2) \((-2i) + (-8i) = \)
3) \((2i) - (6 + 3i) = \)
4) \((4 - 6i) + (-2i) = \)
5) \((-7i) + (4 + 5i) = \)
6) \(10 + (-2 - 6i) = \)
7) \((-3i) - (9 + 2i) = \)
8) \((4 + 6i) - (-3i) = \)

Calculate.

9) \((3 - 2i)(4 - 3i) = \)
10) \((6 + 2i)(3 + 2i) = \)
11) \((8 - i)(4 - 2i) = \)
12) \((2 - 4i)(3 - 5i) = \)
13) \((5 + 6i)(3 + 2i) = \)
14) \((5 + 3i)(9 + 2i) = \)

Simplify.

15) \(\frac{3}{2i} = \)
16) \(\frac{8}{-3i} = \)
17) \(\frac{-9}{2i} = \)
18) \(\frac{2-3i}{-5i} = \)
19) \(\frac{4-5i}{-2i} = \)
20) \(\frac{8+3i}{2i} = \)
Answers – Chapter 15

1) $-12i$  
2) $-10i$  
3) $-6 - i$  
4) $4 - 8i$  
5) $4 - 2i$  
6) $8 - 6i$  
7) $-9 - 5i$  
8) $4 + 9i$  
9) $6 - 17i$  
10) $14 + 18i$  
11) $30 - 20i$  
12) $-14 - 22i$  
13) $3 + 28i$  
14) $39 + 37i$  
15) $-\frac{3i}{2}$  
16) $\frac{8i}{3}$  
17) $-\frac{9i}{2}$  
18) $\frac{3}{5} + \frac{2}{5}i$  
19) $\frac{5}{2} + 2i$  
20) $\frac{3}{2} - 4i$
Math topics that you’ll learn in this chapter:

- ✔ Simplifying Radical Expressions
- ✔ Adding and Subtracting Radical Expressions
- ✔ Multiplying Radical Expressions
- ✔ Rationalizing Radical Expressions
- ✔ Radical Equations
- ✔ Domain and Range of Radical Functions
Simplifying Radical Expressions

- Find the prime factors of the numbers or expressions inside the radical.
- Use radical properties to simplify the radical expression:

\[ \sqrt[n]{x^a} = x^{\frac{a}{n}}, \sqrt[n]{xy} = \sqrt[n]{x} \times \sqrt[n]{y}, \sqrt[n]{\frac{x}{y}} = \frac{\sqrt[n]{x}}{\sqrt[n]{y}}, \text{ and } \sqrt[n]{x} \times \sqrt[n]{y} = \sqrt[n]{xy} \]

Examples:

Example 1. Find the square root of \( \sqrt{144x^2} \).

Solution: Find the factor of the expression \( 144x^2 \): \( 144 = 12 \times 12 \) and \( x^2 = x \times x \), now use radical rule: \( \sqrt[n]{a^n} = a \), Then: \( \sqrt{12^2} = 12 \) and \( \sqrt{x^2} = x \)
Finally: \( \sqrt{144x^2} = \sqrt{12^2} \times \sqrt{x^2} = 12 \times x = 12x \)

Example 2. Write this radical in exponential form. \( \sqrt[3]{x^4} \)

Solution: To write a radical in exponential form, use this rule: \( \sqrt[n]{x^a} = x^{\frac{a}{n}} \)
Then: \( \sqrt[3]{x^4} = x^{\frac{4}{3}} \)

Example 3. Simplify. \( \sqrt{8x^3} \)

Solution: First factor the expression \( 8x^3 \): \( 8x^3 = 2^3 \times x \times x \times x \), we need to find perfect squares: \( 8x^3 = 2^2 \times 2 \times x^2 \times x = 2^2 \times x^2 \times 2x \),
Then: \( \sqrt{8x^3} = \sqrt{2^2} \times \sqrt{x^2} \times \sqrt{2x} \)
Now use radical rule: \( \sqrt[n]{a^n} = a \), Then: \( \sqrt{2^2} \times \sqrt{x^2} \times \sqrt{(2x)} = 2x \times \sqrt{2x} = 2x\sqrt{2x} \)

Example 4. Simplify. \( \sqrt{27a^5b^4} \)

Solution: First factor the expression \( 27a^5b^4 \): \( 27a^5b^4 = 3^3 \times a^5 \times b^4 \), we need to find perfect squares: \( 27a^5b^4 = 3^2 \times 3 \times a^4 \times a \times b^4 \), Then:
\[
\sqrt{27a^5b^4} = \sqrt{3^2 \times a^4 \times b^4 \times \sqrt{3a}}
\]
Now use radical rule: \( \sqrt[n]{a^n} = a \), Then:
\[
\sqrt{3^2 \times a^4 \times b^4 \times \sqrt{3a}} = 3 \times a^2 \times b^2 \times \sqrt{3a} = 3a^2 b^2 \sqrt{3a}
\]
Adding and Subtracting Radical Expressions

- Only numbers and expressions that have the same radical part can be added or subtracted.

- Remember, combining "unlike" radical terms is not possible.

- For numbers with the same radical part, just add or subtract factors outside the radicals.

Examples:

Example 1. Simplify: \(8\sqrt{2} + 4\sqrt{2}\)

Solution: Since we have the same radical parts, then we can add these two radicals: Add like terms: \(8\sqrt{2} + 4\sqrt{2} = 12\sqrt{2}\)

Example 2. Simplify: \(11\sqrt{7} + 6\sqrt{7}\)

Solution: Since we have the same radical parts, then we can add these two radicals: Add like terms: \(11\sqrt{7} + 6\sqrt{7} = 17\sqrt{7}\)

Example 3. Simplify: \(2\sqrt{8} - 2\sqrt{2}\)

Solution: The two radical parts are not the same. First, we need to simplify the \(2\sqrt{8}\). Then: \(2\sqrt{8} = 2\sqrt{4 \times 2} = 2(\sqrt{4})(\sqrt{2}) = 4\sqrt{2}\)

Now, combine like terms: \(2\sqrt{8} - 2\sqrt{2} = 4\sqrt{2} - 2\sqrt{2} = 2\sqrt{2}\)

Example 4. Simplify: \(5\sqrt{27} + 3\sqrt{3}\)

Solution: The two radical parts are not the same. First, we need to simplify the \(5\sqrt{27}\). Then: \(5\sqrt{27} = 5\sqrt{9 \times 3} = 5(\sqrt{9})(\sqrt{3}) = 15\sqrt{3}\)

Now, add: \(5\sqrt{27} + 3\sqrt{3} = 15\sqrt{3} + 3\sqrt{3} = 18\sqrt{3}\)
Multiplying Radical Expressions

To multiply radical expressions:

- Multiply the numbers and expressions outside of the radicals.
- Multiply the numbers and expressions inside the radicals.
- Simplify if needed.

Examples:

Example 1. Evaluate. $2\sqrt{5} \times \sqrt{3}$

Solution: Multiply the numbers outside of the radicals and the radical parts. Then: $2\sqrt{5} \times \sqrt{3} = 2 \times 1 \times \sqrt{5} \times \sqrt{3} = 2\sqrt{15}$

Example 2. Multiply. $3x\sqrt{3} \times 4\sqrt{x}$

Solution: Multiply the numbers outside of the radicals and the radical parts. Then, simplify: $3x\sqrt{3} \times 4\sqrt{x} = (3x \times 4) \times (\sqrt{3} \times \sqrt{x}) = (12x)(\sqrt{3x}) = 12x\sqrt{3x}$

Example 3. Evaluate. $5a\sqrt{5b} \times \sqrt{2b}$

Solution: Multiply the numbers outside of the radicals and the radical parts. Then: $5a\sqrt{5b} \times \sqrt{2b} = 5a \times 1 \times \sqrt{5b} \times \sqrt{2b} = 5a\sqrt{10b^2}$ Simplify: $5a\sqrt{10b^2} = 5a \times \sqrt{10} \times \sqrt{b^2} = 5ab\sqrt{10}$

Example 4. Simplify. $11\sqrt{2x} \times 2\sqrt{8x}$

Solution: Multiply the numbers outside of the radicals and the radical parts. Then, simplify: $11\sqrt{2x} \times 2\sqrt{8x} = (11 \times 2) \times (\sqrt{2x} \times \sqrt{8x}) = (22)(\sqrt{16x^2}) = 22\sqrt{16x^2}$ $\sqrt{16x^2} = 4x$, then: $22\sqrt{16x^2} = 22 \times 4x = 88x$
Chapter 16: Radicals

**Rationalizing Radical Expressions**

- Radical expressions cannot be in the denominator. (number in the bottom)
- To get rid of the radical in the denominator, multiply both numerator and denominator by the radical in the denominator.
- If there is a radical and another integer in the denominator, multiply both numerator and denominator by the conjugate of the denominator.
- The conjugate of \((a + b)\) is \((a - b)\) and vice versa.

**Examples:**

**Example 1.** Simplify \(\frac{6}{\sqrt{2}}\)

**Solution:** Multiply both numerator and denominator by \(\sqrt{2}\). Then:

\[
\frac{6}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{6\sqrt{2}}{2} = 3\sqrt{2}
\]

**Example 2.** Simplify \(\frac{5}{\sqrt{6} - 4}\)

**Solution:** Multiply by the conjugate: \(\frac{\sqrt{6} + 4}{\sqrt{6} + 4} \times \frac{5}{\sqrt{6} - 4} = \frac{5(\sqrt{6} + 4)}{-10}

\(\sqrt{6} - 4)(\sqrt{6} + 4) = -10\), then:

\[
\frac{5}{\sqrt{6} - 4} \times \frac{\sqrt{6} + 4}{\sqrt{6} + 4} = \frac{5(\sqrt{6} + 4)}{-10} = -\frac{1}{2}(\sqrt{6} + 4)
\]

**Example 3.** Simplify \(\frac{2}{\sqrt{3} - 1}\)

**Solution:** Multiply by the conjugate: \(\frac{\sqrt{3} + 1}{\sqrt{3} + 1}\)

\[
\frac{2}{\sqrt{3} - 1} \times \frac{\sqrt{3} + 1}{\sqrt{3} + 1} = \frac{2(\sqrt{3} + 1)}{2} \rightarrow = (\sqrt{3} + 1)
\]

Radical Equations

To solve a radical equation:
- Isolate the radical on one side of the equation.
- Square both sides of the equation to remove the radical.
- Solve the equation for the variable.
- Plugin the answer (answers) into the original equation to avoid extraneous values.

Examples:

Example 1. Solve $\sqrt{x} - 5 = 15$

Solution: Add 5 to both sides: $\sqrt{x} = 20$
Square both sides:
$$(\sqrt{x})^2 = 20^2 \rightarrow x = 400$$
Plugin the value of 400 for $x$ in the original equation and check the answer:
$$x = 400 \rightarrow \sqrt{x} - 5 = \sqrt{400} - 5 = 20 - 5 = 15$$
So, the value of 400 for $x$ is correct.

Example 2. What is the value of $x$ in this equation?
$$2\sqrt{x} + 1 = 4$$

Solution: Divide both sides by 2. Then:
$$2\sqrt{x} + 1 = 4 \rightarrow \frac{2\sqrt{x} + 1}{2} = \frac{4}{2} \rightarrow \sqrt{x} + 1 = 2$$
Square both sides: $$(\sqrt{(x + 1)})^2 = 2^2$$, Then: $x + 1 = 4 \rightarrow x = 3$
Substitute $x$ by 3 in the original equation and check the answer:
$$x = 3 \rightarrow 2\sqrt{x} + 1 = 2\sqrt{3} + 1 = 2\sqrt{4} = 2(2) = 4$$
So, the value of 3 for $x$ is correct.
Domain and Range of Radical Functions

- To find the domain of a radical function, find all possible values of the variable inside radical.

- Remember that having a negative number under the square root symbol is not possible. (For cubic roots, we can have negative numbers)

- To find the range, plugin the minimum and maximum values of the variable inside radical.

Example:

Example 1. Find the domain and range of the radical function. \( y = \sqrt{x - 3} \)

Solution: For domain: Find non-negative values for radicals: \( x - 3 \geq 0 \)
Domain of functions: \( x - 3 \geq 0 \rightarrow x \geq 3 \)
Domain of the function \( y = \sqrt{x - 3} \): \( x \geq 3 \)

For range: The range of a radical function of the form \( c\sqrt{ax + b} + k \) is: \( f(x) \geq k \)
For the function \( y = \sqrt{x - 3} \), the value of \( k \) is 0. Then: \( f(x) \geq 0 \)
Range of the function \( y = \sqrt{x - 3} \): \( f(x) \geq 0 \)

Example 2. Find the domain and range of the radical function. \( y = 5\sqrt{3x + 6} + 4 \)

Solution: For domain: Find non-negative values for radicals: \( 3x + 6 \geq 0 \)
Domain of functions: \( 3x + 6 \geq 0 \rightarrow 3x \geq -6 \rightarrow x \geq -2 \)
Domain of the function \( y = 5\sqrt{3x + 6} + 4 \): \( x \geq -2 \)

For range: The range of a radical function of the form \( c\sqrt{ax + b} + k \) is: \( f(x) \geq k \)
For the function \( y = 5\sqrt{3x + 6} + 4 \), the value of \( k \) is 4. Then: \( f(x) \geq 4 \)
Range of the function \( y = 5\sqrt{3x + 6} + 4 \): \( f(x) \geq 4 \)
Chapter 16: Practices

Simplify.
1) $\sqrt{256y} =$
2) $\sqrt{900} =$
3) $\sqrt{144a^2b} =$
4) $\sqrt{36 \times 9} =$

Simplify.
5) $3\sqrt{5} + 2\sqrt{5} =$
6) $6\sqrt{3} + 4\sqrt{27} =$
7) $5\sqrt{2} + 10\sqrt{18} =$
8) $7\sqrt{2} - 5\sqrt{8} =$

Evaluate.
9) $\sqrt{5} \times \sqrt{3} =$
10) $\sqrt{6} \times \sqrt{8} =$
11) $3\sqrt{5} \times \sqrt{9} =$
12) $2\sqrt{3} \times 3\sqrt{7} =$

Simplify.
13) $\frac{1}{\sqrt{3} - 6} =$
14) $\frac{5}{\sqrt{2} + 7} =$
15) $\frac{\sqrt{3}}{1 - \sqrt{6}} =$
16) $\frac{2}{\sqrt{3} + 5} =$

Solve for $x$.
17) $\sqrt{x} + 2 = 9$
18) $3 + \sqrt{x} = 12$
19) $\sqrt{x} + 5 = 30$
20) $\sqrt{x} - 9 = 27$
21) $10 = \sqrt{x} + 1$
22) $\sqrt{x} + 4 = 3$

Identify the domain and range of each function.
23) $y = \sqrt{x} + 2 - 1$
24) $y = \sqrt{x} + 1$
25) $y = \sqrt{x} - 4$
26) $y = \sqrt{x} - 3 + 1$

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Answers – Chapter 16

1) 16\sqrt{y} 

2) 30 

3) 12a\sqrt{b} 

4) 18 

5) 5\sqrt{5} 

6) 18\sqrt{3} 

7) 35\sqrt{2} 

8) -3\sqrt{2} 

9) \sqrt{15} 

10) \sqrt{48} = 4\sqrt{3} 

11) 9\sqrt{5} 

12) 6\sqrt{21} 

13) -\frac{\sqrt{5}+6}{33} 

14) -\frac{5(\sqrt{2}-7)}{47} 

15) -\frac{\sqrt{3}+3\sqrt{2}}{5} 

16) -\frac{\sqrt{3}−5}{11} 

17) x = 49 

18) x = 81 

19) x = 625 

20) x = 1,296 

21) x = 99 

22) x = 5 

23) x \geq -2, y \geq -1 

24) x \geq -1, y \geq 0 

25) x \geq 4, y \geq 0 

26) x \geq 3, y \geq 1
CHAPTER 17

Logarithms

Math topics that you’ll learn in this chapter:

- Evaluating Logarithms
- Expanding and Condensing Logarithms
- Natural Logarithm
- Solving Logarithmic Equations
Evaluating Logarithms

- Logarithm is another way of writing exponent. \( \log_b y = x \) is equivalent to \( y = b^x \).
- Learn some logarithms rules: \((a > 0, a \neq 0, M > 0, N > 0, \text{ and } k \text{ is a real number.})\)

\[
\begin{align*}
\text{Rule 1: } & \log_a (M \cdot N) = \log_a M + \log_a N \\
\text{Rule 2: } & \log_a \frac{M}{N} = \log_a M - \log_a N \\
\text{Rule 3: } & \log_a (M)^k = k \log_a M \\
\text{Rule 4: } & \log_a a = 1 \\
\text{Rule 5: } & \log_a 1 = 0 \\
\text{Rule 6: } & a^{\log_a k} = k
\end{align*}
\]

Examples:

Example 1. Evaluate: \( \log_2 32 \)

Solution: Rewrite 32 in power base form: \( 32 = 2^5 \), then: \( \log_2 32 = \log_2 (2^5) \)

Use log rule: \( \log_a (M)^k = k \log_a (M) \rightarrow \log_2 (2^5) = 5 \log_2 (2) \)

Use log rule: \( \log_a (a) = 1 \rightarrow \log_2 (2) = 1. \) \( 5 \log_2 (2) = 5 \times 1 = 5 \)

Example 2. Evaluate: \( 3 \log_5 125 \)

Solution: Rewrite 125 in power base form: \( 125 = 5^3 \), then: \( \log_5 125 = \log_5 (5^3) \)

Use log rule: \( \log_a (M)^k = k \log_a (M) \rightarrow \log_5 (5^3) = 3 \log_5 (5) \)

Use log rule: \( \log_a (a) = 1 \rightarrow \log_5 (5) = 1. \) \( 3 \times 3 \log_5 (5) = 3 \times 3 = 9 \)

Example 3. Evaluate: \( \log_3 (3)^5 \)

Solution: Use log rule: \( \log_a (M)^k = k \log_a (M) \rightarrow \log_3 (3)^5 = 5 \log_3 (3) \)

Use log rule: \( \log_a (a) = 1 \rightarrow \log_3 (3) = 1 \rightarrow 5 \times \log_3 (3) = 5 \times 1 = 5 \)
Expanding and Condensing Logarithms

- Using some of properties of logs, (the product rule, quotient rule, and power rule) sometimes we can expand a logarithm expression (expanding) or convert some logarithm expressions into a single logarithm (condensing).

- Let’s review some logarithms properties:

\[
\begin{align*}
\ln a^b &= b \\
\ln a &= 0 \\
\ln a a &= 1 \\
\ln (a . b) &= \ln a + \ln b \\
\ln a \div b &= \ln a - \ln b \\
\ln x^p &= p \ln x
\end{align*}
\]

Examples:

Example 1. Expand this logarithm. \(\ln (3 \times 5) =\)

Solution: Use log rule: \(\ln (a . b) = \ln a + \ln b\)
Then: \(\ln (3 \times 5) = \ln 3 + \ln 5\)

Example 2. Condense this expression to a single logarithm. \(\ln 2 - \ln 7\)

Solution: Use log rule: \(\ln a - \ln b = \ln \frac{a}{b}\)
Then: \(\ln 2 - \ln 7 = \ln \frac{2}{7}\)

Example 3. Expand this logarithm. \(\ln \left(\frac{1}{7}\right) =\)

Solution: Use log rule: \(\ln a \div b = -\ln a x\)
Then: \(\ln \left(\frac{1}{7}\right) = -\ln 7\)
Natural Logarithms

- A natural logarithm is a logarithm that has a special base of the mathematical constant $e$, which is an irrational number approximately equal to 2.71.
- The natural logarithm of $x$ is generally written as $\ln x$, or $\log_e x$.

Examples:

Example 1. Expand this natural logarithm. $\ln 4x^2 =$

Solution: Use log rule: $\log_a (xy) = \log_a x + \log_a y$
Then: $\ln 4x^2 = \ln 4 + \ln x^2$. Now, use log rule: $\log_a (M)^k = k \cdot \log_a (M) \rightarrow ln 4 + \ln x^2 = \ln 4 + 2 \ln x$

Example 2. Condense this expression to a single logarithm. $\ln x - \log_e 2y$

Solution: Use log rule: $\log_a x - \log_a y = \log_a \frac{x}{y}$
Then: $\ln x - \log_e 2y = \ln \frac{x}{2y}$

Example 3. Solve this equation for $x$: $e^x = 6$

Solution: If $f(x) = g(x)$, then: $\ln(f(x)) = \ln(g(x)) \rightarrow \ln(e^x) = \ln(6)$
Use log rule: $\log_a x^b = b \log_a x \rightarrow \ln(e^x) = x \ln(e) \rightarrow x\ln(e) = \ln(6)$
$\ln(e) = 1$, then: $x = \ln(6)$

Example 4. Solve this equation for $x$: $\ln(4x - 2) = 1$

Solution: Use log rule: $a = \log_b (b^a) \rightarrow 1 = \ln(e^1) = \ln(e) \rightarrow \ln(4x - 2) = \ln (e)$
When the logs have the same base: $\log_b (f(x)) = \log_b (g(x)) \rightarrow f(x) = g(x)$
$\ln(4x - 2) = \ln(e)$, then: $4x - 2 = e \rightarrow x = \frac{e+2}{4}$

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Solving Logarithmic Equations

To solve a logarithm equation:
- Convert the logarithmic equation to an exponential equation when it’s possible. (If no base is indicated, the base of the logarithm is 10)
- Condense logarithms if you have more than one log on one side of the equation.
- Plug in the answers back into the original equation and check to see if the solution works.

Examples:

Example 1. Find the value of x in this equation. \( \log_2(36 - x^2) = 4 \)

Solution: Use log rule: \( \log_b x = \log_b y \), then: \( x = y \)
We can write number 4 as a logarithm: \( 4 = \log_2(2^4) \)
Then: \( \log_2(36 - x^2) = \log_2(2^4) = \log_2 16 \)
Then: \( 36 - x^2 = 16 \) → \( 36 - 16 = x^2 \) → \( x^2 = 20 \) → \( x = \pm \sqrt{20} = \pm 2\sqrt{5} \)
You can plug in back the solutions into the original equation to check your answer.
\( x = \sqrt{20} \) → \( \log_2(36 - \sqrt{20}^2) = 4 \) → \( \log_2(36 - 20) = 4 \) → \( \log_2 16 = 4 \)
\( x = -\sqrt{20} \) → \( \log_2(36 - (-\sqrt{20})^2) = 4 \) → \( \log_2(36 - 20) = 4 \) → \( \log_2 16 = 4 \)
Both solutions work in the original equation.

Example 2. Find the value of x in this equation. \( \log(5x + 2) = \log(3x - 1) \)

Solution: When the logs have the same base: \( f(x) = g(x) \), then:
\( \ln(f(x)) = \ln(g(x)) \), \( \log(5x + 2) = \log(3x - 1) \) → \( 5x + 2 = 3x - 1 \) →
\( 5x + 2 - 3x + 1 = 0 \) → \( 2x + 3 = 0 \) → \( 2x = -3 \) → \( x = -\frac{3}{2} \)
Verify Solution: \( \log(5x + 2) = \log \left( 5\left(-\frac{3}{2}\right) + 2 \right) = \log (-5.5) \)
Logarithms of negative numbers are not defined. Therefore, there is no solution for this equation.
Chapter 17: Practices

Expand each logarithm.

1) \( \log_b(2 \times 9) = \)

2) \( \log_b(5 \times 7) = \)

3) \( \log_b(xy) = \)

4) \( \log_b(2x^2 \times 3y) = \)

Evaluate each logarithm.

5) \( 2 \log_9(9) = \)

6) \( 3 \log_2(8) = \)

7) \( 2 \log_5(125) = \)

8) \( \log_{100}(1) = \)

9) \( \log_{10}(100) = \)

10) \( 3 \log_4(16) = \)

11) \( \frac{1}{2} \log_3(81) = \)

12) \( \log_7(343) = \)

Reduce the following expressions to simplest form.

13) \( e^{ln4+ln5} = \)

14) \( e^{ln(\frac{3}{e})} = \)

15) \( e^{ln2+ln7} = \)

16) \( 6 \ln(e^5) = \)

Find the value of the variables in each equation.

17) \( \log_3(8x) = 3 \rightarrow x = \) ___

18) \( \log_4(2x) = 5 \rightarrow x = \) ___

19) \( \log_4(5x) = 0 \rightarrow x = \) ___

20) \( \log_4(4x) = \log_5 \rightarrow x = \) ___
Answers – Chapter 17

1) \( \log_b 2 + 2 \log_b 3 \)  
2) \( \log_b 5 + \log_b 7 \)  
3) \( \log_b x + \log_b y \)  
4) \( \log_b 2 + \log_b x^2 + \log_b 3 + \log_b y \)  
5) 2  
6) 9  
7) 6  
8) 0  
9) 2  
10) 6

11) 2  
12) 3  
13) 20  
14) \( \frac{9}{e} \)  
15) 14  
16) 30  
17) \( \frac{27}{8} \)  
18) 512  
19) \( \frac{1}{5} \)  
20) \( \frac{5}{4} \)
Math topics that you’ll learn in this chapter:

- Circumference and Area of Circles
- Arc length and sector Area
- Equation of a Circle
- Finding the Center and the Radius of Circles
Circumference and Area of Circles

- In a circle, variable $r$ is usually used for the radius and $d$ for diameter.

- *Area of a circle* = $\pi r^2$ ($\pi$ is about 3.14)

- *Circumference of a circle* = $2\pi r$

Examples:

**Example 1.** Find the area of this circle.

**Solution:**
Use area formula: $Area = \pi r^2$
$r = 8 \text{ in} \rightarrow Area = \pi (8)^2 = 64\pi$, $\pi = 3.14$
Then: $Area = 64 \times 3.14 = 200.96 \text{ in}^2$

**Example 2.** Find the Circumference of this circle.

**Solution:**
Use Circumference formula: $Circumference = 2\pi r$
$r = 5 \text{ cm} \rightarrow Circumference = 2\pi (5) = 10\pi$
$\pi = 3.14$ Then: $Circumference = 10 \times 3.14 = 31.4 \text{ cm}$

**Example 3.** Find the area of the circle.

**Solution:**
Use area formula: $Area = \pi r^2$
$r = 5 \text{ in}$, then: $Area = \pi (5)^2 = 25\pi$, $\pi = 3.14$
Then: $Area = 25 \times 3.14 = 78.5$
Arc Length and Sector Area

- To find the area of a sector of a circle, use this formula:
  \[
  \text{Area of a sector} = \pi r^2 \left( \frac{\theta}{360} \right),
  \]
  \( r \) is the radius of the circle and \( \theta \) is the central angle of the sector.

- To find the arc of a sector of a circle, use this formula:
  \[
  \text{Arc of a sector} = \frac{\theta}{180} \pi r
  \]

Examples:

Example 1. Find the length of the arc. Round your answers to the nearest tenth.

\((\pi = 3.14), r = 20 \text{ cm}, \theta = 30^\circ\)

Solution: Use this formula: \[
\text{Length of the sector} = \left( \frac{\theta}{180} \right) \pi r = \left( \frac{30}{180} \right) \pi (20) = \left( \frac{1}{6} \right) \pi (20) = \left( \frac{20}{6} \right) \times 3.14 \approx 10.5 \text{ cm}
\]

Example 2. Find the area of the sector. \((\pi = 3.14) \ r = 6 \text{ ft}, \theta = 70^\circ\)

Solution: Use this formula: \[
\text{area of a sector} = \pi r^2 \left( \frac{\theta}{360} \right)
\]
Area of the sector = \[
\pi r^2 \left( \frac{\theta}{360} \right) = (3.14)(6^2) \left( \frac{70}{360} \right) = 21.98 \text{ ft}^2
\]

Example 3. Find the length of the arc. \((\pi = 3.14) \ r = 3 \text{ ft}, \theta = \frac{\pi}{3}\)

Solution: \[
\theta = \frac{\pi}{3} \rightarrow \frac{\pi}{3} \times \frac{180}{\pi} = 60^\circ
\]
Length of the sector = \[
\left( \frac{60}{180} \right) \pi (3) = \left( \frac{1}{3} \right) \pi (3) = 1 \times 3.14 = 3.14 \text{ cm}
\]
Equation of a Circle

- Equation of circles in standard form: \((x - h)^2 + (y - k)^2 = r^2\)

  Center: \((h, k)\), Radius: \(r\)

- Equation of circles in general form: \(x^2 + y^2 + Ax + By + C = 0\)

Examples:

Write the standard form equation of each circle.

Example 1. \(x^2 + y^2 - 4x - 6y + 9 = 0\)

Solution: The standard form of circle equation is: \((x - h)^2 + (y - k)^2 = r^2\)
where the radius of the circle is \(r\), and it’s centered at \((h, k)\).
First, move the loose number to the right side: \(x^2 + y^2 - 4x - 6y = -9\)
Group \(x\)-variables and \(y\)-variables together: \((x^2 - 4x) + (y^2 - 6y) = -9\)
Convert \(x\) to square form:
\((x^2 - 4x + 4) + y^2 - 6y = -9 + 4 \rightarrow (x - 2)^2 + (y^2 - 6y) = -9 + 4\)
Convert \(y\) to square form:
\((x - 2)^2 + (y^2 - 6y + 9) = -9 + 4 + 9 \rightarrow (x - 2)^2 + (y - 3)^2 = 4\)
Then, the equation of the circle in standard form is: \((x - 2)^2 + (y - 3)^2 = 2^2\)

Example 2. The center of the circle is at \((-2, -10)\), and its radius is 5.

Solution: \((x - h)^2 + (y - k)^2 = r^2\) is the circle equation with a radius \(r\),
centered at \((h, k)\). So, \(h = -2, k = -10\) and \(r = 5\)
Then, the equation of the circle is: \((x - (-2))^2 + (y - (-10))^2 = (5)^2\)
Finding the Center and the Radius of Circles

To find the center and the radius of a circle using the equation of the circle:

1. Write the equation of the circle in standard form: \((x - h)^2 + (y - k)^2 = r^2\),
2. The center of the circle is at \((h, k)\), and its radius is \(r\).

Examples:

Identify the center and the radius of each circle:

Example 1. \(x^2 + y^2 - 4x + 3 = 0\)

Solution: \((x - h)^2 + (y - k)^2 = r^2\) is the circle equation with a radius \(r\), centered at \((h, k)\).
Rewrite \(x^2 + y^2 - 4x + 3 = 0\) in the standard form:
\[x^2 + y^2 - 4x + 3 = 0 \rightarrow (x - 2)^2 + y^2 = 1^2\]
Then, the center is at: \((2, 0)\) and \(r = 1\)

Example 2. \(8x + x^2 + 10y = 8 - y^2\)

Solution: Rewrite the equation in standard form:
\[8x + x^2 + 10y = 8 - y^2 \rightarrow (x - (-4))^2 + (y - (-5))^2 = 7^2\]
Then, the center is at \((-4, -5)\) and the radius is 7.
Chapter 18: Practices

Complete the table below. \((\pi = 3.14)\)

<table>
<thead>
<tr>
<th></th>
<th>Radius</th>
<th>Diameter</th>
<th>Circumference</th>
<th>Area</th>
</tr>
</thead>
<tbody>
<tr>
<td>Circle 1</td>
<td>3 inches</td>
<td>6 inches</td>
<td>18.84 inches</td>
<td>28.26 square inches</td>
</tr>
<tr>
<td>Circle 2</td>
<td></td>
<td></td>
<td>43.96 meters</td>
<td></td>
</tr>
<tr>
<td>Circle 3</td>
<td>8 ft</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Circle 4</td>
<td></td>
<td></td>
<td></td>
<td>78.5 square miles</td>
</tr>
</tbody>
</table>

Find the length of each arc. Round your answers to the nearest hundredth.

2) \(r = 4\) cm, \(\theta = 28^\circ\) → arc =

3) \(r = 6\) ft, \(\theta = 30^\circ\) → arc =

4) \(r = 8\) ft, \(\theta = 40^\circ\) → arc =

5) \(r = 12\) cm, \(\theta = 34^\circ\) → arc =

Write the standard form equation of each circle.

6) \(x^2 + y^2 - 4x + 2y - 4 = 0\) →

7) \(x^2 + y^2 - 8x + 6y - 11 = 0\) →

8) \(x^2 + y^2 - 10x - 12y + 12 = 0\) →

9) \(x^2 + y^2 + 12x - 6y - 19 = 0\) →

10) \(x^2 + y^2 - 6x + 8y + 24 = 0\) →

Identify the center and radius of each circle.

11) \((x + 1)^2 + (y - 2)^2 = 5\) → Center: (___,___) Radius: _____

12) \((x - 5)^2 + (y + 10)^2 = 4\) → Center: (___,___) Radius: _____

13) \(x^2 + (y - 3)^2 = 8\) → Center: (___,___) Radius: _____

14) \((x - 1)^2 + y^2 = 9\) → Center: (___,___) Radius: _____

15) \(x^2 + y^2 = 16\) → Center: (___,___) Radius: _____

16) \((x + 1)^2 + (y + 6)^2 = 10\) → Center: (___,___) Radius: _____
Answers – Chapter 18

<table>
<thead>
<tr>
<th>Circle</th>
<th>Radius</th>
<th>Diameter</th>
<th>Circumference</th>
<th>Area</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
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<td>6 inches</td>
<td>18.84 inches</td>
<td>28.26 square inches</td>
</tr>
<tr>
<td>2</td>
<td>7 meters</td>
<td>14 meters</td>
<td>43.96 meters</td>
<td>153.86 square meters</td>
</tr>
<tr>
<td>3</td>
<td>4 ft</td>
<td>8 ft</td>
<td>25.12 ft</td>
<td>50.24 square ft</td>
</tr>
<tr>
<td>4</td>
<td>5 miles</td>
<td>10 miles</td>
<td>31.4 miles</td>
<td>78.5 square miles</td>
</tr>
</tbody>
</table>

2) 1.95 cm

3) 3.14 ft

4) 5.58 ft

5) 7.12 cm

6) \((x - 2)^2 + (y - (1))^2 = 3^2\)

7) \((x - 4)^2 + (y - (3))^2 = 6^2\)

8) \((x - 5)^2 + (y - 6)^2 = 7^2\)

9) \((x - (6))^2 + (y - 3)^2 = 8^2\)

10) \((x - 3)^2 + (y - (4))^2 = 1^2\)

11) Center: \((-1, 2)\), Radius: \(\sqrt{5}\)

12) Center: \((5, -10)\), Radius: 2

13) Center: \((0, 3)\), Radius: \(2\sqrt{2}\)

14) Center: \((1, 0)\), Radius: 3

15) Center: \((0, 0)\), Radius: 4

16) Center: \((-1, -6)\), Radius: \(\sqrt{10}\)
Math topics that you’ll learn in this chapter:

- ✔ Simplifying Complex Fractions
- ✔ Graphing Rational Expressions
- ✔ Adding and Subtracting Rational Expressions
- ✔ Multiplying Rational Expressions
- ✔ Dividing Rational Expressions
- ✔ Rational Equations
Simplifying Complex Fractions

- Convert mixed numbers to improper fractions.
- Simplify all fractions.
- Write the fraction in the numerator of the main fraction line then write division sign (÷) and the fraction of the denominator.
- Use normal method for dividing fractions.
- Simplify as needed.

Example:

Example 1. Simplify: $\frac{\frac{3}{5}}{\frac{25}{16}}$

Solution: First, simplify the denominator: $\frac{2}{25} - \frac{5}{16} = -\frac{93}{400}$.

Then: $\frac{\frac{3}{5}}{\frac{25}{16}} = \frac{\frac{3}{5}}{-\frac{93}{400}}$; Now, write the complex fraction using the division sign:

$\frac{\frac{3}{5}}{-\frac{93}{400}} = \frac{3}{5} ÷ (-\frac{93}{400})$. Use the dividing fractions rule: Keep, Change, Flip (keep the first fraction, change the division sign to multiplication, flip the second fraction)

$\frac{3}{5} \times \frac{400}{-93} = -\frac{240}{93} = -\frac{80}{31} = 2\frac{18}{31}$

Example 2. Simplify: $\frac{\frac{2 + \frac{1}{3}}{\frac{5 + \frac{1}{3}}{9 + \frac{1}{3}}}}{\frac{5}{9} + \frac{1}{3}}$

Solution: First, simplify the numerator: $\frac{2}{5} ÷ \frac{1}{3} = \frac{6}{5}$, then, simplify the denominator: $\frac{5}{9} + \frac{1}{3} = \frac{8}{9}$, Now, write the complex fraction using the division sign (÷): $\frac{\frac{2 + \frac{1}{3}}{\frac{5 + \frac{1}{3}}{9 + \frac{1}{3}}}}{\frac{5}{9} + \frac{1}{3}} = \frac{\frac{6}{5}}{\frac{8}{9}} \times \frac{9}{8} = \frac{9}{5} \div \frac{8}{9}$, Use the dividing fractions rule: (Keep, Change, Flip)

$\frac{9}{5} ÷ \frac{8}{9} = \frac{9}{5} \times \frac{9}{8} = \frac{54}{40} = \frac{27}{20} = 1\frac{7}{20}$
Graphing Rational Expressions

- A rational expression is a fraction in which the numerator and/or the denominator are polynomials. Examples: \( \frac{1}{x} \), \( x^2 - x + 2 \), \( \frac{m^2 + 6m - 5}{m - 2m} \)
- To graph a rational function:
  - Find the vertical asymptotes of the function if there is any. (Vertical asymptotes are vertical lines which correspond to the zeroes of the denominator. The graph will have a vertical asymptote at \( x = a \) if the denominator is zero at \( x = a \) and the numerator isn’t zero at \( x = a \))
  - Find the horizontal or slant asymptote. (If the numerator has a bigger degree than the denominator, there will be a slant asymptote. To find the slant asymptote, divide the numerator by the denominator using either long division or synthetic division.)
  - If the denominator has a bigger degree than the numerator, the horizontal asymptote is the \( x \)-axes or the line \( y = 0 \). If they have the same degree, the horizontal asymptote equals the leading coefficient (the coefficient of the largest exponent) of the numerator divided by the leading coefficient of the denominator.
  - Find intercepts and plug in some values of \( x \) and solve for \( y \), then graph the function.

Example:

**Example 1.** Graph rational function. \( f(x) = \frac{x^2 - x + 2}{x - 1} \)

**Solution:** First, notice that the graph is in two pieces. Most rational functions have graphs in multiple pieces. Find \( y - intercept \) by substituting zero for \( x \) and solving for \( y \) (\( f(x) \)): \( x = 0 \) \( \rightarrow \) \( y = \frac{x^2 - x + 2}{x - 1} = \frac{0^2 - 0 + 2}{0 - 1} = -2 \),

\( y - intercept: (0, -2) \)

Asymptotes of \( \frac{x^2 - x + 2}{x - 1} \): Vertical: \( x = 1 \), Slant asymptote: \( y = 2x + 1 \) (divide the numerator by the denominator). After finding the asymptotes, you can plug in some values for \( x \) and solve for \( y \). Here is the sketch for this function.
Adding and Subtracting Rational Expressions

For adding and subtracting rational expressions:

- Find least common denominator (LCD).
- Write each expression using the LCD.
- Add or subtract the numerators.
- Simplify as needed.

Examples:

Example 1. Solve. \( \frac{4}{2x+3} + \frac{x-2}{2x+3} = \)

**Solution:** The denominators are equal. Then, use fractions addition rule:

\[
\frac{a}{c} \pm \frac{b}{c} = \frac{a \pm b}{c} \rightarrow \frac{4}{2x+3} + \frac{x-2}{2x+3} = \frac{4+(x-2)}{2x+3} = \frac{x+2}{2x+3}
\]

Example 2. Solve. \( \frac{x+4}{x-5} + \frac{x-4}{x+6} = \)

**Solution:** Find the least common denominator of \((x - 5)\) and \((x + 6)\): \((x - 5)(x + 6)\)

Then:

\[
\frac{x+4}{x-5} + \frac{x-4}{x+6} = \frac{(x+4)(x+6) + (x-4)(x-5)}{(x-5)(x+6)} = \frac{(x+4)(x+6)+(x-4)(x-5)}{(x+6)(x-5)}
\]

Expand: \((x + 4)(x + 6) + (x - 4)(x - 5) = 2x^2 + x + 44\)

Then:

\[
\frac{(x+4)(x+6)+(x-4)(x-5)}{(x + 6)(x-5)} = \frac{2x^2+x+44}{(x+6)(x-5)} = \frac{2x^2+x+44}{x^2+x-30}
\]

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Multiplying Rational Expressions

- Multiplying rational expressions is the same as multiplying fractions. First, multiply numerators and then multiply denominators. Then, simplify as needed.

Examples:

Example 1. Solve: $\frac{x + 6}{x - 1} \times \frac{x - 1}{5} = $

Solution: Multiply numerators and denominators: $\frac{a}{b} \times \frac{c}{d} = \frac{a \times c}{b \times d}$

$$\frac{x + 6}{x - 1} \times \frac{x - 1}{5} = \frac{(x + 6)(x - 1)}{5(x - 1)}$$

Cancel the common factor: $(x - 1)$

Then: $\frac{(x + 6)(x - 1)}{5(x - 1)} = \frac{x + 6}{5}$

Example 2. Solve: $\frac{x - 2}{x + 3} \times \frac{2x + 6}{x - 2} = $

Solution: Multiply numerators and denominators: $\frac{x - 2}{x + 3} \times \frac{2x + 6}{x - 2} = \frac{(x - 2)(2x + 6)}{(x + 3)(x - 2)}$

Cancel the common factor: $\frac{(x - 2)(2x + 6)}{(x + 3)(x - 2)} = \frac{2x + 6}{x + 3}$

Factor $2x + 6 = 2(x + 3)$

Then: $\frac{2(x + 3)}{(x + 3)} = 2$
Dividing Rational Expressions

- To divide rational expressions, use the same method we use for dividing fractions. (Keep, Change, Flip)
- Keep the first rational expression, change the division sign to multiplication, and flip the numerator and denominator of the second rational expression. Then, multiply numerators and multiply denominators. Simplify as needed.

Examples:

Example 1. Solve: \( \frac{x^2+5x+6}{3x^2+3x} \)

Solution: Use fractions division rule: \( \frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c} = \frac{a \times d}{b \times c} \)

\[
\frac{x + 2}{3x} \div \frac{x^2 + 5x + 6}{3x^2 + 3x} = \frac{x + 2}{3x} \times \frac{3x^2 + 3x}{x^2 + 5x + 6} = \frac{(x + 2)(3x^2 + 3x)}{(3x)(x^2 + 5x + 6)}
\]

Now, factorize the expressions \( 3x^2 + 3x \) and \( x^2 + 5x + 6 \). Then:

\( 3x^2 + 3x = 3x(x + 1) \) and \( x^2 + 5x + 6 = (x + 2)(x + 3) \)

Simplify: \( \frac{(x+2)(3x^2+3x)}{(3x)(x^2+5x+6)} = \frac{(x+2)(3x)(x+1)}{(3x)(x+2)(x+3)} \), cancel common factors. Then:

\[
\frac{(x+2)(3x)(x+1)}{(3x)(x+2)(x+3)} = \frac{x+1}{x+3}
\]

Example 2. Solve: \( \frac{5x}{x + 3} \div \frac{x}{2x + 6} \)

Solution: Use fractions division rule: \( \frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c} = \frac{a \times d}{b \times c} \)

Then: \( \frac{5x}{x + 3} \div \frac{x}{2x + 6} = \frac{5x}{x + 3} \times \frac{2x + 6}{x} = \frac{5x(2x + 6)}{x(x+3)} = \frac{5x\times2(x+3)}{x(x+3)} \)

Cancel common factor: \( \frac{5x\times2(x+3)}{x(x+3)} = \frac{10x(x+3)}{x(x+3)} = 10 \)
Rational Equations

For solving rational equations, we can use following methods:

- **Converting to a common denominator:** In this method, you need to get a common denominator for both sides of the equation. Then, make numerators equal and solve for the variable.

- **Cross-multiplying:** This method is useful when there is only one fraction on each side of the equation. Simply multiply the first numerator by the second denominator and make the result equal to the product of the second numerator and the first denominator.

**Examples:**

**Example 1.** Solve \( \frac{x-2}{x+1} = \frac{x+4}{x-2} \)

**Solution:** Use cross multiply method: if \( \frac{a}{b} = \frac{c}{d} \), then: \( a \times d = b \times c \)

\[
\frac{x-2}{x+1} = \frac{x+4}{x-2} \rightarrow (x-2)(x-2) = (x+4)(x+1)
\]

Expand: \((x-2)^2 = x^2 - 4x + 4 \) and \((x+4)(x+1) = x^2 + 5x + 4\),

Then: \( x^2 - 4x + 4 = x^2 + 5x + 4 \), Now, simplify: \( x^2 - 4x = x^2 + 5x \), subtract both sides \((x^2 + 5x)\),

Then: \( x^2 - 4x - (x^2 + 5x) = x^2 + 5x - (x^2 + 5x) \rightarrow -9x = 0 \rightarrow x = 0 \)

**Example 2.** Solve \( \frac{2x}{x-3} = \frac{2x + 2}{2x-6} \)

**Solution:** Multiply the numerator and denominator of the rational expression on the left by 2 to get a common denominator \((2x - 6)\), \( \frac{2(2x)}{2(x-3)} = \frac{4x}{2x-6} \)

Now, the denominators on both side of the equation are equal. Therefore, their numerators must be equal too.

\[
\frac{4x}{2x-6} = \frac{2x + 2}{2x-6} \rightarrow 4x = 2x + 2 \rightarrow 2x = 2 \rightarrow x = 1
\]
Chapter 19: Practices

Simplify each expression.

1) \( \frac{2}{5} \cdot \frac{4}{7} = \)

2) \( \frac{6}{5} \cdot \frac{x + 2}{x + 3} = \)

3) \( \frac{1}{x + 4} - \frac{2}{x + 1} = \)

4) \( \frac{x}{3} \cdot \frac{5}{x} = \)

Graph rational expressions.

5) \( f(x) = \frac{x^2}{5x + 6} \)

6) \( f(x) = \frac{x^2 + 8x + 10}{x + 5} \)

Simplify each expression.

7) \( \frac{5}{x + 2} + \frac{x - 1}{x + 2} = \)

8) \( \frac{6}{x + 5} - \frac{5}{x + 5} = \)

9) \( \frac{7}{4x + 10} + \frac{x - 5}{4x + 10} = \)
Simplify each expression.

10) \( \frac{x+1}{x+5} \times \frac{x+6}{x+1} = \)

11) \( \frac{x+4}{x+9} \times \frac{x+9}{x+3} = \)

12) \( \frac{x+8}{x} \times \frac{2}{x+8} = \)

13) \( \frac{x+5}{x+1} \times \frac{x^2}{x+5} = \)

14) \( \frac{x-3}{x+2} \times \frac{2x+4}{x+4} = \)

15) \( \frac{x-6}{x+3} \times \frac{2x+6}{2x} = \)

Solve.

16) \( \frac{5x}{4} \div \frac{5}{2} = \)

17) \( \frac{8}{3x} \div \frac{24}{x} = \)

18) \( \frac{3x}{x+4} \div \frac{x}{3x+12} = \)

19) \( \frac{2}{5x} \div \frac{16}{10x} = \)

20) \( \frac{36x}{5} \div \frac{4}{3} = \)

21) \( \frac{15x^2}{6} \div \frac{5x}{14} = \)

Solve each equation.

22) \( \frac{1}{8x^2} = \frac{1}{4x^2} - \frac{1}{x} \rightarrow x = \_\_\_\_\_

23) \( \frac{1}{x} + \frac{1}{9x} = \frac{5}{36} \rightarrow x = \_\_\_\_\_

24) \( \frac{32}{2x^2} + 1 = \frac{8}{x} \rightarrow x = \_\_\_\_\_

25) \( \frac{1}{x-5} = \frac{4}{x-5} + 1 \rightarrow x = \_\_\_\_

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Answers – Chapter 19

1) \( \frac{7}{10} \)
2) \( \frac{18x}{17} \)
3) \( \frac{x^2-2x-3}{x^2+4x-5} \)
4) \( \frac{4x^2}{3x-20} \)
5) 
6) 

7) \( \frac{x+4}{x+2} \)
8) \( \frac{1}{x+5} \)
9) \( \frac{x+2}{4x+10} \)
10) \( \frac{x+6}{x+5} \)
11) \( \frac{x+4}{x+3} \)
12) \( \frac{2}{x} \)
13) \( \frac{x^2}{x+1} \)
14) \( \frac{2(x-3)}{x+4} \)
15) \( \frac{x-6}{x} \)
16) \( \frac{x}{2} \)
17) \( \frac{1}{9} \)
18) 9
19) \( \frac{1}{4} \)
20) \( \frac{27x}{5} \)
21) 7x
22) \( x = \frac{1}{8} \)
23) \( x = 8 \)
24) \( x = 4 \)
25) \( x = 2 \)
Math topics that you’ll learn in this Chapter:

- Angle and Angle Measure
- Trigonometric Functions
- Coterminal Angles and Reference Angles
- Evaluating Trigonometric Functions
- Missing Sides and Angles of a Right Triangle
Angle and Angle Measure

- To convert degrees to radians, use this formula:
  \[ \text{Radians} = \text{Degrees} \times \frac{\pi}{180} \]

- To convert radians to degrees, use this formula:
  \[ \text{Degrees} = \text{Radians} \times \frac{\pi}{180} \]

Examples:

Example 1. Convert 160 degrees to radian.

Solution: Use this formula: \( \text{Radians} = \text{Degrees} \times \frac{\pi}{180} \)

\[
\text{Radians} = 160 \times \frac{\pi}{180} = \frac{160\pi}{180} = \frac{8\pi}{9}
\]

Example 2. Convert radian measure \( \frac{3\pi}{4} \) to degree measure.

Solution: Use this formula: \( \text{Degrees} = \text{Radians} \times \frac{180}{\pi} \)

\[
\text{Radians} = \frac{3\pi}{4} \times \frac{180}{\pi} = \frac{540\pi}{4\pi} = 135
\]

Example 3. Convert 150 degrees to radian.

Solution: Use this formula: \( \text{Radians} = \text{Degrees} \times \frac{\pi}{180} \)

\[
\text{Radians} = 150 \times \frac{\pi}{180} = \frac{150\pi}{180} = \frac{5\pi}{6}
\]

Example 4. Convert radian measure \( \frac{3\pi}{4} \) to degree measure.

Solution: Use this formula: \( \text{Degrees} = \text{Radians} \times \frac{180}{\pi} \)

\[
\text{Radians} = \frac{2\pi}{3} \times \frac{180}{\pi} = \frac{360\pi}{3\pi} = 120
\]
Trigonometric Functions

- Trigonometric functions refer to the relation between the sides and angles of a right triangle. There are 6 trigonometric functions:
  - Sine (sin), Cosine (cos), Tangent (tan), Secant (sec), Cosecant (csc), and Cotangent (cot)
  - The three main trigonometric functions:
    SOH – CAH – TOA, \( \sin \theta = \frac{\text{opposite}}{\text{hypotenuse}}, \cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}}, \tan \theta = \frac{\text{opposite}}{\text{adjacent}} \)
  - The reciprocal trigonometric functions:
    \( \csc x = \frac{1}{\sin x}, \sec x = \frac{1}{\cos x}, \cot \theta = \frac{1}{\tan x} \)
  - Learn common trigonometric functions:

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Examples:

Find each trigonometric function.

Example 1. \( \sin 120^\circ \)

**Solution:** Use the following property: \( \sin(x) = \cos(90^\circ - x) \)
\( \sin 120^\circ = \cos(90^\circ - 120^\circ) = \cos(-30^\circ) = \frac{\sqrt{3}}{2} \)

Example 2. \( \tan 120^\circ \)

**Solution:** Use the following property: \( \tan(x) = \frac{\sin(x)}{\cos(x)} \)
\( \tan(120) = \frac{\sin(120)}{\cos(120)} = \frac{\frac{\sqrt{3}}{2}}{-\frac{1}{2}} = -\sqrt{3} \)
Coterminal Angles and Reference Angles

- Coterminal angles are equal angles.
- To find a Coterminal of an angle, add or subtract 360 degrees (or $2\pi$ for radians) to the given angle.
- Reference angle is the smallest angle that you can make from the terminal side of an angle with the $x$-axis.

Examples:

Example 1. Find a positive and a negative Coterminal angle to angle $65^\circ$.

Solution:

$65^\circ - 360^\circ = -295^\circ$
$65^\circ + 360^\circ = 425^\circ$

$-295^\circ$ and $425^\circ$ are Coterminal with angle $65^\circ$.

Example 2. Find positive and negative Coterminal angles to angle $\frac{\pi}{2}$.

Solution:

$\frac{\pi}{2} + 2\pi = \frac{5\pi}{2}$
$\frac{\pi}{2} - 2\pi = -\frac{3\pi}{2}$

Example 3. Find a positive and a negative Coterminal angle to angle $80^\circ$.

Solution:

$80^\circ - 360^\circ = -280^\circ$
$80^\circ + 360^\circ = 440^\circ$

$-280^\circ$ and $440^\circ$ are Coterminal with angle $80^\circ$. 

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Evaluating Trigonometric Functions

- **Step 1:** Find the reference angle. (It is the smallest angle that you can make from the terminal side of an angle with the \( x \)-axis.)

- **Step 2:** Determine the quadrant of the function. Depending on the quadrant in which the function lies, the answer will be either positive or negative.

- **Step 3:** Find the trigonometric function of the reference angle.

**Examples:**

**Example 1.** Find the exact value of trigonometric function. \( \tan \frac{5\pi}{4} \)

**Solution:** Rewrite the angle for \( \frac{5\pi}{4} \):
\[
\tan \frac{5\pi}{4} = \tan \left( \frac{4\pi + \pi}{4} \right) = \tan \left( \pi + \frac{1}{4}\pi \right)
\]
Use the periodicity of \( \tan \): \( \tan(x + \pi \cdot k) = \tan(x) \)
\[
\tan \left( \pi + \frac{1}{4}\pi \right) = \tan \left( \frac{1}{4}\pi \right) = 1
\]

**Example 2.** Find the exact value of trigonometric function. \( \cos 225^\circ \)

**Solution:** First, recall that \( \cos (225^\circ) \) is in the third quadrant and cosine is negative in the third quadrant.
The reference angle of \( 225^\circ \) is \( 45^\circ \). Therefore, \( \cos 225^\circ = -\cos 45^\circ \)
\[
\cos 45^\circ = \frac{\sqrt{2}}{2} \]
Then, \( -\cos 45^\circ = -\frac{\sqrt{2}}{2} \)

**Example 3.** Find the exact value of trigonometric function. \( \sin \frac{7\pi}{6} \)

**Solution:** Rewrite the \( \sin \frac{7\pi}{6} \).
\[
\sin \frac{7\pi}{6} = \sin \left( \frac{\pi}{6} + \pi \right) = \cos \left( \frac{\pi}{6} \right) \] (complementary arcs)

Trig Table of Special Arcs gives: \( \cos \frac{\pi}{6} = \frac{\sqrt{3}}{2} \)
Missing Sides and Angles of a Right Triangle

- By using three main trigonometric functions (Sine, Cosine or Tangent), we can find an unknown side in a right triangle when we have one length, and one angle (apart from the right angle).

- A right triangle with Adjacent and Opposite sides and Hypotenuse is shown below.

- Recall the three main trigonometric functions:

  \[
  \text{SOH} - \text{CAH} - \text{TOA}, \quad \sin \theta = \frac{\text{opposite}}{\text{hypotenuse}}, \quad \cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}}, \quad \tan \theta = \frac{\text{opposite}}{\text{adjacent}}
  \]

- To find missing angles, use inverse of trigonometric functions (examples: \(\sin^{-1}\), \(\cos^{-1}\), and \(\tan^{-1}\))

Examples:

Example 1. Find side AC in the following triangle. Round your answer to the nearest tenth.

Solution: \(\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}}\). \(\sin 50^\circ = \frac{AC}{6} \rightarrow 6 \times \sin 50^\circ = AC\),

Now use a calculator to find \(\sin 50^\circ\).

\(\sin 50^\circ \approx 0.766\)

\(AC = 6 \times 0.766 = 4.596\), rounding to the nearest tenth: \(4.596 \approx 4.6\)

Example 2. Find the value of \(x\) in the following triangle.

Solution: \(\cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}}\) \(\rightarrow \cos x = \frac{10}{14} = \frac{5}{7}\)

Use a calculator to find inverse cosine:

\(\cos^{-1}\left(\frac{5}{7}\right) = 44.41^\circ \approx 44^\circ\)

Then: \(x = 44\)
Chapter 20: Trigonometric Functions

Chapter 20: Practices

Convert each degree measure into radians.

1) \(135^\circ = \)  
2) \(80^\circ = \)  
3) \(270^\circ = \)  
4) \(92^\circ = \)  

Evaluate.

5) \(\sin 90^\circ = \)  
6) \(\sin -330^\circ = \)  
7) \(\tan -30^\circ = \)  
8) \(\cot \frac{2\pi}{3} = \)  
9) \(\tan \frac{\pi}{3} = \)  
10) \(\sin \frac{2\pi}{6} = \)  

Find a positive and a negative Coterterminal angle for each angle.

11) \(140^\circ = \)  
   Positive =  
   Negative =  
12) \(-165^\circ = \)  
   Positive =  
   Negative =  
13) \(190^\circ = \)  
   Positive =  
   Negative =  
14) \(\frac{5\pi}{4} = \)  
   Positive =  
   Negative =  
15) \(\frac{2\pi}{9} = \)  
   Positive =  
   Negative =  
16) \(-\frac{7\pi}{9} = \)  
   Positive =  
   Negative =  

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Find the exact value of each trigonometric function.

17) \( \cos 180^\circ = \) 
20) \( \sin \frac{\pi}{4} = \) 
18) \( \cos -270^\circ = \) 
21) \( \csc 330^\circ = \) 
19) \( \tan 225^\circ = \) 
22) \( \tan -120^\circ = \) 

Find the value of \( x \) in each triangle.

23)____
24)____
25)____

26)____
27)____
28)____
Answers – Chapter 20

1) \( \frac{3}{4} \pi \)

2) \( \frac{4}{9} \pi \)

3) \( \frac{3}{2} \pi \)

4) \( \frac{23}{45} \pi \)

5) 1

6) \( \frac{1}{2} \)

7) \( -\frac{\sqrt{3}}{3} \)

8) \( -\frac{\sqrt{3}}{3} \)

9) \( \sqrt{3} \)

10) \( \frac{\sqrt{3}}{2} \)

11) Positive = 500\(^{\circ}\), Negative = -220\(^{\circ}\)

12) Positive = 195\(^{\circ}\), Negative = -525\(^{\circ}\)

13) Positive = 550\(^{\circ}\), Negative = -170\(^{\circ}\)

14) Positive = \( \frac{13\pi}{4} \), Negative = \( -\frac{3\pi}{4} \)

15) Positive = \( \frac{20\pi}{9} \), Negative = \( -\frac{16\pi}{9} \)

16) Positive = \( \frac{11\pi}{9} \), Negative = \( -\frac{25\pi}{9} \)

17) -1

18) 0

19) 1

20) \( \frac{\sqrt{2}}{2} \)

21) -2

22) \( \sqrt{3} \)

23) 18

24) 7.8

25) 63

26) 4.5

27) 12.45

28) 14
Time to Test

Time to refine your skill with a practice examination

Take an ALEKS Mathematics practice test to simulate the test day experience. After you've finished, score your test using the answers and explanations section.

Before You Start

❖ You'll need a pencil and scratch papers to take the test.
❖ For these practice tests, don’t time yourself. Spend time as much as you need.
❖ After you’ve finished the test, review the answer key to see where you went wrong.

Good luck!
ALEKS Mathematics Practice Test 1

2021 - 2022

Total number of questions: 30
Total time (Calculator): No time limit

Calculators are permitted for ALEKS Math Test.

(On a real ALEKS test, there is an onscreen calculator to use.)
1) If \( f(x) = 4x - 2 \) and \( g(x) = x^2 - x \), then find \( \frac{f}{g}(x) \).

2) A bank is offering 4.5% simple interest on a savings account. If you deposit $12,000, how much interest will you earn in two years?

3) If the ratio of home fans to visiting fans in a crowd is 3:2 and all 24,000 seats in a stadium are filled, how many visiting fans are in attendance?

4) If the interior angles of a quadrilateral are in the ratio 2:3:3:4, what is the measure of the largest angle?

5) If the area of a circle is 49 square meters, what is its diameter?
6) The length of a rectangle is $\frac{5}{4}$ times its width. If the width is 20, what is the perimeter of this rectangle?

7) In the figure below, line $A$ is parallel to line $B$. What is the value of angle $x$?

8) An angle is equal to one ninth of its supplement. What is the measure of that angle?

9) What is the value of $y$ in the following system of equations?

$$2x + 5y = 11$$
$$4x - 2y = -14$$

10) Last week 25,000 fans attended a football match. This week three times as many bought tickets, but one sixth of them cancelled their tickets. How many are attending this week?
11) If $\sin A = \frac{1}{3}$ in a right triangle and the angle $A$ is an acute angle, then what is $\cos A$?

12) In the standard $(x, y)$ coordinate system plane, what is the area of the circle with the following equation?

$$(x + 2)^2 + (y - 4)^2 = 25$$

13) Convert 580,000 to scientific notation.

14) The ratio of boys to girls in a school is 2:3. If there are 500 students in a school, how many boys are in the school.

15) If 150% of a number is 75, then what is 80% of that number?
16) If $A = \begin{bmatrix} -1 & 2 \\ 1 & -2 \end{bmatrix}$ and $B = \begin{bmatrix} 3 \\ -2 \\ 1 \\ 3 \end{bmatrix}$, then $2A - B =$

17) What is the solution of the following inequality?

$$|x - 2| \geq 4$$

18) If $\tan x = \frac{8}{15}$, then $\sin x =$?

19) $(x^6)^{\frac{7}{6}}$ equal to?

20) What are the zeroes of the function $f(x) = x^3 + 5x^2 + 6x$?
21) If $x + \sin^2 a + \cos^2 a = 3$, then $x = ?$

22) If $\sqrt{5x} = \sqrt{y}$, then $x = \ ?$

23) The average weight of 18 girls in a class is 55 kg and the average weight of 32 boys in the same class is 62 kg. What is the average weight of all the 50 students in that class?

24) What is the value of the expression $5(x - 2y) + (2 - x)^2$ when $x = 3$ and $y = -3$ ?

25) Sophia purchased a sofa for $530.40. The sofa is regularly priced at $631. What was the percent discount Sophia received on the sofa?
26) If one angle of a right triangle measures 60°, what is the sine of the other acute angle?

27) Simplify \( \frac{4 - 3i}{-4i} \)?

28) The average of five consecutive numbers is 40. What is the smallest number?

29) What is the slope of a line that is perpendicular to the line \( 4x - 2y = 14 \)?

30) If \( f(x) = 2x^3 + 4 \) and \( g(x) = \frac{1}{x} \), what is the value of \( f(g(x)) \)?

This is the end of Practice Test 1.
AULEKS Mathematics Practice Test 2

2021 - 2022

Total number of questions: 30
Total time (Calculator): No time limit

Calculators are permitted for AULEKS Math Test.

(On a real AULEKS test, there is an onscreen calculator to use.)
1) How many tiles of 8 cm² is needed to cover a floor of dimension 6 cm by 24 cm?

2) What is the area of a square whose diagonal is 8 cm?

3) What is the value of x in the following figure?

4) What is the value of y in the following system of equation?

\[\begin{align*}
3x - 4y &= -20 \\
-x + 2y &= 10
\end{align*}\]

5) How long does a 420–miles trip take moving at 50 miles per hour (mph)?
6) When 40% of 60 is added to 12% of 600, the resulting number is:

7) What is the solution of the following inequality?

\[ |x - 10| \leq 3 \]

8) In the following figure, ABCD is a rectangle, and E and F are points on AD and DC, respectively. The area of \( \Delta BED \) is 16, and the area of \( \Delta BDF \) is 18. What is the perimeter of the rectangle?

![Rectangle Diagram]

9) If a tree casts a 24–foot shadow at the same time that a 3 feet yardstick casts a 2–foot shadow, what is the height of the tree?

10) A ladder leans against a wall forming a 60° angle between the ground and the ladder. If the bottom of the ladder is 30 feet away from the wall, how long is the ladder?
11) Simplify. \(2x^2 + 3y^5 - x^2 + 2z^3 - 2y^2 + 2x^3 - 2y^5 + 5z^3\)

12) In five successive hours, a car traveled 40 km, 45 km, 50 km, 35 km and 55 km. In the next five hours, it traveled with an average speed of 50 km per hour. Find the total distance the car traveled in 10 hours.

13) In the following figure, ABCD is a rectangle. If \(a = \sqrt{3}\), and \(b = 2a\), find the area of the shaded region. (the shaded region is a trapezoid)

14) 6 liters of water are poured into an aquarium that's 15cm long, 5cm wide, and 90cm high. How many centimeters will the water level in the aquarium rise due to this added water? (1 liter of water = 1,000 cm\(^3\))

15) If a box contains red and blue balls in ratio of 2:3, how many red balls are there if 90 blue balls are in the box?
16) A chemical solution contains 4% alcohol. If there is 24 ml of alcohol, what is the volume of the solution?

17) If \( \frac{3x}{16} = \frac{x-1}{4} \), then \( x = \) 

18) Simplify \((-5 + 9i)(3 + 5i)\).

19) If \( \theta \) is an acute angle and \( \sin \theta = \frac{4}{5} \) then \( \cos \theta = \)

20) If 60% of \( x \) equal to 30% of 20, then what is the value of \( (x + 5)^2 \)?
21) A boat sails 40 miles south and then 30 miles east. How far is the boat from its start point?

22) What is the value of $x$ in the following equation? $\log_4(x + 2) - \log_4(x - 2) = 1$

23) A number is chosen at random from 1 to 25. Find the probability of not selecting a composite number.

24) Find AC in the following triangle. Round your answer to the nearest tenth.

25) If $y = 4ab + 3b^3$, what is $y$ when $a = 2$ and $b = 3$?
26) If \( f(x) = 5 + x \) and \( g(x) = -x^2 - 1 - 2x \), then find \((g - f)(x)\).

27) If cotangent of an angle \( \beta \) is 1, then the tangent of angle \( \beta \) is ...

28) When point \( A (10, 3) \) is reflected over the \( y \)-axis to get the point \( B \), what are the coordinates of point \( B \)?

29) What is the average of circumference of figure \( A \) and area of figure \( B \)? \((\pi = 3)\)

30) If \( f(x) = 2x^3 + 5x^2 + 2x \) and \( g(x) = -2 \), what is the value of \( f(g(x)) \)?

This is the end of Practice Test 2.
ALEKS Mathematics
Practice Tests
Answers and Explanations
ALEKS Mathematics Practice Test 1
Answers and Explanations

1) The answer is $\frac{4x^2 - 2}{x^2 - x}$
\[
\left( \frac{f}{g} \right)(x) = \frac{f(x)}{g(x)} = \frac{4x - 2}{x^2 - x}
\]

2) The answer is 1,080

   Use simple interest formula: $I = prt \ (I = \text{interest}, \ p = \text{principal}, \ r = \text{rate}, \ t = \text{time})$, $I = (12,000)(0.045)(2) = 1,080$

3) The answer is 9,600

   Number of visiting fans: $\frac{2 \times 24,000}{5} = 9,600$

4) The answer is 120°

   The sum of all angles in a quadrilateral is 360 degrees. Let $x$ be the smallest angle in the quadrilateral. Then the angles are: $2x, 3x, 3x, 4x$, $2x + 3x + 3x + 4x = 360 \rightarrow 12x = 360 \rightarrow x = 30$, The angles in the quadrilateral are: 60°, 90°, 90°, and 120°

5) The answer is $\frac{14\sqrt{\pi}}{\pi}$

   Formula for the area of a circle is: $A = \pi r^2$, Using 49 for the area of the circle we have: $49 = \pi r^2$, Let’s solve for the radius ($r$).
   
   $\frac{49}{\pi} = r^2 \rightarrow r = \sqrt{\frac{49}{\pi}} = \frac{7}{\sqrt{\pi}} = \frac{7\sqrt{\pi}}{\pi}$
   
   Then, the diameter of the circle is: $d = 2r \rightarrow d = 2 \times \frac{7\sqrt{\pi}}{\pi} = \frac{14\sqrt{\pi}}{\pi}$

6) The answer is 90

   Length of the rectangle is: $\frac{5}{4} \times 20 = 25$, perimeter of rectangle is: $2 \times (20 + 25) = 90$
7) **The answer is 135°**

The angle $x$ and 45 are complementary angles. Therefore: $x + 45 = 180 \rightarrow x = 180° - 45° = 135°$

8) **The answer is 18**

The sum of supplement angles is 180. Let $x$ be that angle. Therefore, $x + 9x = 180$

$10x = 180$, divide both sides by 10: $x = 18$

9) **The answer is 3**

Solving Systems of Equations by Elimination: Multiply the first equation by $(−2)$, then add it to the second equation.

$−2(2x + 5y = 11) \Rightarrow −4x − 10y = −22$

$4x − 2y = −14 \Rightarrow 4x − 2y = −14 \Rightarrow −12y = −36 \Rightarrow y = 3$

10) **The answer is 62,500**

Three times of 25,000 is 75,000. One sixth of them cancelled their tickets. One sixth of 75,000 equals 12,500 ($\frac{1}{6} \times 72,000 = 12,500$). 62,500 ($72,000 − 12,000 = 62,500$) fans are attending this week.

11) **The answer is $\sqrt{8}$**

$sinA = \frac{1}{3} \Rightarrow$ Since $sin\theta = \frac{\text{o}pp\text{os}e}{\text{h}yp\text{oten}u\text{se}}$, we have the following right triangle. Then:

$c = \sqrt{3^2 - 1^2} = \sqrt{9 - 1} = \sqrt{8}$, $cosA = \frac{\sqrt{8}}{3}$

12) **The answer is $25\pi$**

The equation of a circle in standard form is: $(x - h)^2 + (y - k)^2 = r^2$, where $r$ is the radius of the circle. In this circle the radius is 5. $r^2 = 25 \rightarrow r = 5$, $(x + 2)^2 + (y - 4)^2 = 25$

Area of a circle: $A = \pi r^2 = \pi (5)^2 = 25\pi$

13) **The answer is $5.8 \times 10^5$**

580,000 = $5.8 \times 10^5$
14) **The answer is 200**

The ratio of boy to girls is 2:3. Therefore, there are 2 boys out of 5 students. To find the answer, first divide the total number of students by 5, then multiply the result by 2.

\[ 500 \div 5 = 100 \Rightarrow 100 \times 2 = 200 \]

15) **The answer is 40**

First, find the number. Let \( x \) be the number. Write the equation and solve for \( x \).

150\% of a number is 75, then:

\[ 1.5 \times x = 75 \Rightarrow x = 75 \div 1.5 = 50 \]

80\% of 50 is: \( 0.8 \times 50 = 40 \)

16) **The answer is \([\begin{pmatrix} -5 \\ 3 \\ -7 \end{pmatrix}]\)**

First, find \( 2A \). \[ A = \begin{pmatrix} -1 & 2 \\ 1 & -2 \end{pmatrix} \]; \[ 2A = 2 \times \begin{pmatrix} -1 & 2 \\ 1 & -2 \end{pmatrix} = \begin{pmatrix} -2 & 4 \\ 2 & -4 \end{pmatrix} \]. Now, solve for \[ 2A - B = \begin{pmatrix} -2 & 4 \\ 2 & -4 \end{pmatrix} - \begin{pmatrix} 3 & 1 \\ -2 & 3 \end{pmatrix} = \begin{pmatrix} -5 & 3 \\ 4 & -7 \end{pmatrix} \]

17) **The answer is \( x \geq 6 \cup x \leq -2 \)**

\[ x - 2 \geq 4 \rightarrow x \geq 4 + 2 \rightarrow x \geq 6, \text{ Or } x - 2 \leq -4 \rightarrow x \leq -4 + 2 \rightarrow x \leq -2 \]

Then, solution is: \( x \geq 6 \cup x \leq -2 \)

18) **The answer is \( \frac{8}{17} \)**

\[ \tan = \frac{\text{opposite}}{\text{adjacent}} \], and \( \tan x = \frac{8}{15} \); therefore, the opposite side of the angle \( x \) is 8 and the adjacent side is 15. Let’s draw the triangle.

Using Pythagorean theorem, we have:

\[ a^2 + b^2 = c^2 \rightarrow 8^2 + 15^2 = c^2 \rightarrow 64 + 225 = c^2 \rightarrow c = 17, \sin x = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{8}{17} \]

19) **The answer is \( x^{\frac{21}{4}} \)**

\[ (x^6)^{\frac{7}{3}} = x^6 \times x^{\frac{7}{3}} = x^{\frac{21}{3}} = x^{\frac{21}{4}} \]
20) **The answer are 0, −2, −3**

First factor the function: \( f(x) = x^3 + 5x^2 + 6x = x(x + 2)(x + 3) \). To find the zeros, \( f(x) \) should be zero. \( f(x) = x(x + 2)(x + 3) = 0 \), Therefore, the zeros are: \( x = 0 \), \( (x + 2) = 0 \Rightarrow x = −2 \), \( (x + 3) = 0 \Rightarrow x = −3 \)

21) **The answer is 2**

\( \sin^2 a + \cos^2 a = 1 \), then: \( x + 1 = 3 \), \( x = 2 \)

22) **The answer is \( \frac{y}{5} \)**

Solve for \( x \). \( \sqrt{5x} = \sqrt{y} \). Square both sides of the equation: \( (\sqrt{5x})^2 = (\sqrt{y})^2 \); \( 5x = y \); \( x = \frac{y}{5} \)

23) **The answer is 59.48**

The sum of the weight of all girls is:

\( 18 \times 55 = 990 \text{ kg} \)

The sum of the weight of all boys is: \( 32 \times 62 = 1984 \text{ kg} \), The sum of the weight of all students is: \( 990 + 1984 = 2974 \text{ kg} \). \( average = \frac{2974}{50} = 59.48 \)

24) **The answer is 46**

Plug in the value of \( x \) and \( y \). \( x = 3 \) and \( y = −3 \)

\( 5(x - 2y) + (2 - x)^2 = 5(3 - 2(-3)) + (2 - 3)^2 = 5(3 + 6) + (-1)^2 = 45 + 1 = 46 \)

25) **The answer is 16%**

The question is this: 530.40 is what percent of 631? Use percent formula:

\( part = \frac{\text{percent}}{100} \times \text{whole} \)

\( 530.40 = \frac{\text{percent}}{100} \times 631 \Rightarrow 530.40 = \frac{\text{percent} \times 631}{100} \Rightarrow 53,040 = \text{percent} \times 631 \Rightarrow \text{percent} = \frac{53,040}{631} = 84 \). 530.40 is 84% of 631. Therefore, the discount is: \( 100% - 84% = 16% \)
26) The answer is $\frac{1}{2}$

The relationship among all sides of right triangle $30^\circ - 60^\circ - 90^\circ$ is provided in the following triangle: Sine of $30^\circ$ equals to:

$$\frac{\text{opposite}}{\text{hypotenuse}} = \frac{x}{2x} = \frac{1}{2}$$

27) The answer is $\frac{3}{4} + i$

To simplify the fraction, multiply both numerator and denominator by $i$.

\[
\frac{4-3i}{-4i} \times \frac{i}{i} = \frac{4i-3i^2}{-4i^2}, \quad i^2 = -1,
\]

Then:

\[
\frac{4i-3i^2}{-4(-1)} = \frac{4i+3}{4} = \frac{4i}{4} + \frac{3}{4} = \frac{3}{4} + i
\]

28) The answer is 38

Let $x$ be the smallest number. Then, these are the numbers: $x, x + 1, x + 2, x + 3, x + 4$

\[
\text{average} = \frac{\text{sum of terms}}{\text{number of terms}} \Rightarrow 40 = \frac{x + (x+1) + (x+2) + (x+3) + (x+4)}{5} \Rightarrow 200 = 5x + 10 \Rightarrow 190 = 5x \Rightarrow x = 38
\]

29) The answer is $-\frac{1}{2}$

The equation of a line in slope intercept form is: $y = mx + b$, Solve for $y$. $4x - 2y = 14 \Rightarrow -2y = 14 - 4x \Rightarrow y = (14 - 4x) / (-2) \Rightarrow y = 2x - 7$, The slope is 2.

The slope of the line perpendicular to this line is:

\[
m_1 \times m_2 = -1 \Rightarrow 2 \times m_2 = -1 \Rightarrow m_2 = -\frac{1}{2}
\]

30) The answer is $\frac{2}{x^3} + 4$

\[
f(g(x)) = 2 \times \left(\frac{1}{x}\right)^3 + 4 = \frac{2}{x^3} + 4
\]
ALEKS Mathematics Practice Test 2
Answers and Explanations

1) The answer is 18

The area of the floor is: $6\,cm \times 24\,cm = 144\,cm^2$.
The number of tiles needed $= 144 \div 8 = 18$

2) The answer is 32

The diagonal of the square is 8. Let $x$ be the side.
Use Pythagorean Theorem: $a^2 + b^2 = c^2$
$x^2 + x^2 = 8^2 \Rightarrow 2x^2 = 8^2 \Rightarrow 2x^2 = 64 \Rightarrow x^2 = 32$
$\Rightarrow x = \sqrt{32}$
The area of the square is: $\sqrt{32} \times \sqrt{32} = 32$

3) The answer is 145

$x = 20 + 125 = 145$

4) The answer is 5

Solve the system of equations by elimination method.
$3x - 4y = -20$
$-x + 2y = 10$
Multiply the second equation by 3, then add it to the first equation.

$3x - 4y = -20$
$3(-x + 2y = 10) \Rightarrow -3x + 6y = 30$
$\Rightarrow$ add the equations $2y = 10 \Rightarrow y = 5$

5) The answer is 8.4 hours

Use distance formula: $Distance = Rate \times time \Rightarrow 420 = 50 \times T$, divide both sides by
$50$. $420 \div 50 = T \Rightarrow T = 8.4$ hours. Change hours to minutes for the decimal part. $0.4\,hours = 0.4 \times 60 = 24\,minutes$.

6) The answer is 96
40% of 60 equals to: $0.40 \times 60 = 24$, 12% of 600 equals to: $0.12 \times 600 = 72$
40% of 60 is added to 12% of 600: $24 + 72 = 96$

7) The answer is $7 \leq x \leq 13$

$|x - 10| \leq 3 \rightarrow -3 \leq x - 10 \leq 3 \rightarrow -3 + 10 \leq x - 10 + 10 \leq 3 + 10 \rightarrow 7 \leq x \leq 13$

8) The answer is 40

The area of $\triangle BED$ is 16, then: $\frac{4 \times AB}{2} = 16 \rightarrow 4 \times AB = 32 \rightarrow AB = 8$

The area of $\triangle BDF$ is 18, then: $\frac{3 \times BC}{2} = 18 \rightarrow 3 \times BC = 36 \rightarrow BC = 12$

The perimeter of the rectangle is $= 2 \times (8 + 12) = 40$

9) The answer is 36 ft

Write a proportion and solve for $x$. \( \frac{3}{2} = \frac{x}{24} \Rightarrow 2x = 3 \times 24 \Rightarrow x = 36 \text{ ft} \)

10) The answer is 60 ft

The relationship among all sides of special right triangle $30^\circ - 60^\circ - 90^\circ$ is provided in this triangle:
In this triangle, the opposite side of $30^\circ$ angle is half of the hypotenuse.

Draw the shape of this question:
The ladder is the hypotenuse. Therefore, the ladder is 60 ft.

11) The answer is $y^5 + 2x^3 + 7z^3 + x^2 - 2y^2$

\[
2x^2 + 3y^5 - x^2 + 2z^3 - 2y^2 + 2x^3 - 2y^5 + 5z^3 \\
= 2x^2 - x^2 + 2x^3 - 2y^2 + 3y^5 - 2y^5 + 2z^3 + 5z^3 \\
= x^2 + 2x^3 - 2y^2 + y^5 + 7z^3 \\
\]

Write the expression in standard form:
\[
x^2 + 2x^3 - 2y^2 + y^5 + 7z^3 = y^5 + 2x^3 + 7z^3 + x^2 - 2y^2
\]
12) The answer is 475

Add the first 5 numbers. \(40 + 45 + 50 + 35 + 55 = 225\)
To find the distance traveled in the next 5 hours, multiply the average by number of hours.
\(Distance = Average \times Rate = 50 \times 5 = 250\). Add both numbers.
\(250 + 225 = 475\)

13) The answer is \(4\sqrt{3}\)

Based on triangle similarity theorem: \(\frac{a}{a+b} = \frac{c}{3} \rightarrow \frac{c}{a+b} = \frac{3\sqrt{3}}{3\sqrt{3}} = 1 \rightarrow \) area of shaded region is: \(\frac{c+3}{2} \times b = \frac{4}{2} \times 2\sqrt{3} = 4\sqrt{3}\)

14) The answer is 80cm

\(One \ liter = 1,000 \ cm^3 \rightarrow 6 \ liters = 6,000 \ cm^3; \)
\(6,000 = 15 \times 5 \times h \rightarrow h = \frac{6,000}{75} = 80cm\)

15) The answer is 60

\(\frac{2}{3} \times 90 = 60\)

16) The answer is 600 ml

4% of the volume of the solution is alcohol. Let \(x\) be the volume of the solution.
Then: \(4\% \ of \ x = 24 \ ml \Rightarrow 0.04 \times x = 24 \Rightarrow x = 24 \div 0.04 = 600\)

17) The answer is 4

Solve for \(x\). \(\frac{3x}{16} = \frac{x-1}{4}\). Multiply the second fraction by \(4\). \(\frac{3x}{16} = \frac{4(x-1)}{4 \times 4}\). Tow denominators are equal. Therefore, the numerators must be equal.\(3x = 4x - 4, 0 = x - 4, 4 = x\)

18) The answer is \(-60 + 2i\)

We know that: \(i = \sqrt{-1} \Rightarrow i^2 = -1\)
\((-5 + 9i)(3 + 5i) = -15 - 25i + 27i + 45i^2 = -15 + 2i - 45 = -60 + 2i\)
19) The answer is $\frac{3}{5}$

\[ \sin \theta = \frac{4}{5} \Rightarrow \text{we have following triangle, then} \]
\[ c = \sqrt{5^2 - 4^2} = \sqrt{25 - 16} = \sqrt{9} = 3, \cos \theta = \frac{3}{5} \]

20) The answer is 225

\[ 0.6x = (0.3) \times 20 \rightarrow x = 10 \rightarrow (x + 5)^2 = (15)^2 = 225 \]

21) The answer is 50 miles

Use the information provided in the question to draw the shape.

Use Pythagorean Theorem: $a^2 + b^2 = c^2$
\[ 40^2 + 30^2 = c^2 \Rightarrow 1600 + 900 = c^2 \Rightarrow 2500 = c^2 \]
\[ \Rightarrow c = 50 \]

22) The answer is $\frac{10}{3}$

METHOD ONE

\[ \log_4(x + 2) - \log_4(x - 2) = 1, \text{ Add } \log_4(x - 2) \text{ to both sides} \]
\[ \log_4(x + 2) - \log_4(x - 2) + \log_4(x - 2) = 1 + \log_4(x - 2) \]
\[ \log_4(x + 2) = 1 + \log_4(x - 2) \]

Apply logarithm rule: $a = \log_b(b^a) \Rightarrow 1 = \log_4(4^1) = \log_4(4)$

then: $\log_4(x + 2) = \log_4(4) + \log_4(x - 2)$

Logarithm rule: $\log_c(a) + \log_c(b) = \log_c(ab)$

then: $\log_4(4) + \log_4(x - 2) = \log_4(4(x - 2))$
\[ \log_4(x + 2) = \log_4(4(x - 2)) \]

When the logs have the same base: $\log_b(f(x)) = \log_b(g(x)) = f(x) = g(x)$
\[ (x + 2) = 4(x - 2), x = \frac{10}{3} \]

METHOD TWO

We know that: $\log_a b - \log_a c = \log_a \frac{b}{c}$ and $\log_a b = c \Rightarrow b = a^c$

Then: $\log_4(x + 2) - \log_4(x - 2) = \log_4 \frac{x + 2}{x - 2} = 1 \Rightarrow \frac{x + 2}{x - 2} = 4^1 = 4 \Rightarrow x + 2 = 4(x - 2)$
\[ \Rightarrow x + 2 = 4x - 8 \Rightarrow 4x - x = 8 + 2 \Rightarrow 3x = 10 \Rightarrow x = \frac{10}{3} \]
23) The answer is $\frac{2}{5}$

Set of number that are not composite between 1 and 25:

$$A = \{1, 2, 3, 5, 7, 11, 13, 17, 19, 23\}$$

$$\text{Probability} = \frac{\text{number of desired outcomes}}{\text{number of total outcomes}} = \frac{10}{25} = \frac{2}{5}$$

24) The answer is 3.9

$$\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}} \Rightarrow \sin 40^\circ = \frac{AC}{6} \times \sin 40^\circ = AC,$$

Now use a calculator to find $\sin 40^\circ$. $\sin 40^\circ \approx 0.642 \Rightarrow AC \approx 3.9$

25) The answer is 105

$$y = 4ab + 3b^3$$. Plug in the values of $a$ and $b$ in the equation: $a = 2$ and $b = 3$

$$y = 4(2)(3) + 3(3)^3 = 24 + 3(27) = 24 + 81 = 105$$

26) The answer is $-x^2 - 3x - 6$

$$(g - f)(x) = g(x) - f(x) = (-x^2 - 1 - 2x) - (5 + x)$$

$$-x^2 - 1 - 2x - 5 - x = -x^2 - 3x - 6$$

27) The answer is 1

$$\tan \beta = \frac{1}{\cot \beta} = \frac{1}{1} = 1$$

28) The answer is ($-10, 3$)

When points are reflected over $y$-axis, the value of $y$ in the coordinates doesn’t change and the sign of $x$ changes. Therefore, the coordinates of point $B$ is ($-10, 3$).

29) The answer is 60

Perimeter of figure $A$ is: $2\pi r = 2\pi \frac{20}{2} = 20\pi = 20 \times 3 = 60$

Area of figure $B$ is: $5 \times 12 = 60$, Average $= \frac{60 + 60}{2} = \frac{120}{2} = 60$

30) The answer is 0

$$g(x) = -2$$, then $f(g(x)) = f(-2) = 2(-2)^3 + 5(-2)^2 + 2(-2) = -16 + 20 - 4 = 0$
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